Biped locomotion created by a controller based on Zero-Moment Point (ZMP) known as reliable control method looks different from human’s walking on the view point that ZMP-based walking does not include falling state, and it’s like monkey walking because of knee-bended walking profiles. However, the walking control that does not depend on ZMP is vulnerable to turnover. Therefore, keeping the event-driven walking of dynamical motion stable is important issue for realization of human-like natural walking. In this research, a walking model of humanoid robot including slipping, bumping, surface-contacting and line-contacting of foot is discussed, and its dynamical equation is derived by the Extended NE method. In this paper we introduce the humanoid model which including the slipping foot and verify the model.

Keywords: humanoid, slipping, friction, bipedal, dynamical

1. Introduction

Human beings have acquired an ability of stable bipedal walking in evolving repetitions so far. From a view point of making a stable controller for the bipedal walking based on knowledge of control theory is not easy, because of the dynamics with high nonlinearity and coupled interactions between state variables with high dimensions. Therefore how to simplify the complicated walking dynamics to help construct stable walking controller has been studied intensively.

To avoiding complications in dealing directly with true dynamics (without approximation), inverted pendulum has been used frequently for making a stable controller [1–3], simplifying the calculations to determine input torque. Further, linear approximation having the humanoid being represented by simple inverted pendulum enables researchers to realize stable gait through well-known control strategy [4–6].

Our research has begun from a view point of [7, 8] as aiming to describing gait’s dynamics as correctly as possible, including point-contacting state of foot and toe, slipping of the foot and bumping. We discuss the dynamics of whole-body humanoid that contains head, waist and arms. And that we think more important is that the dimension of dynamical equation will change depending on the walking gait’s varieties, which has been discussed by [9] about concerning one legged hopping robot. In fact, this kind of dynamics with the dimension number of state variables varying by the result of its dynamical time transitions that are out of the arena of control theory that discusses how to control a system with fixed states’ number. Further the tipping over motion has been called as non-holonomic dynamics that includes a joint without inputting torque, i.e., free joint.

Meanwhile, landing of the heel or the toe of lifting leg in the air to the ground makes a geometrical contact. Based on [10], We derive the dynamics of humanoid which is simulated as a serial-link manipulator including constraint motion and slipping motion by using the Extended Newton-Euler Method [11].

The conventional method of the NE could be applied to a robot having an open loop serial linkage structure, but the motion of hand was limited to motions without contacting external world. The NE method has not been formulated although it was very important for a robot that works under a premise it must be contacted with the environment when the robot was doing some grinding work or assembling work. For this point, the extended NE method was proposed in [11] that is as same as the research of [12], in terms of that the constraints are strictly satisfied. Meanwhile, the constraint force which can be included in the iterative calculation of the NE method by calculating the constraint force by a substitution method [13].

In this research that based on [14, 15], a walking model of humanoid robot including slipping, bumping, surface-contacting and point-contacting of foot is discussed, and its dynamical equation is derived by the NE method. Especially the common consideration of the free-leg model [16–18] is without any slipping. This research is different from the conventional consideration, that the nonlinear friction which includes the static/kinetic friction will be discussed in walking model of humanoid model.

In this paper, a dynamical model of humanoid including influences of nonlinearity caused by stick-slip [19–21] motions, which are derived from the nonlinear friction between the floor and humanoid’s feet, will be introduced.
Furthermore, as the preparation of walking simulation, some gaits models and their transition conditions will also be introduced.

2. Dynamical Walking Model by Newton-Euler Method

2.1. Forward Kinematical Calculations

We discuss a biped robot whose definition is depicted in Fig. 1. Table 1 indicates length \( l_i \) [m], mass \( m_i \) [kg] of links and coefficient of joints’ viscous friction \( d_i \) [N-m-s/rad], which are decided based on [22]. This model is simulated as a serial-link manipulator having ramifications and represents rigid whole body – feet including torso, arms and so on – by 17 degree-of-freedom. Though motion of legs is restricted in sagittal plane, it generates varieties of walking gait sequences since the robot has flat-sole feet and kicking torque. In this paper, one foot including link-1 is defined as “supporting-leg” and another foot including link-7 is defined as “free-leg” (“contacting-leg” when the free-leg contacts with floor) according to the walking state.

In this paper, we derive the equation of motion following by NE formulation [22, 23]. So we must consider the structure of the supporting-leg with two situations. When the supporting-leg is constituted by prismatic joint. We will switch the equations as the following.

Then if the supporting-leg is constituted by prismatic joint. We will switch the equations as the following.

\[ i^\dot{\mathbf{p}}_i = \dot{l}_{i-1}R_i \{ -1 \mathbf{p}_{i-1} + l_{i-1} \mathbf{\omega}_{i-1} \times \mathbf{p}_{i-1} \} \]  

\[ i^\ddot{\mathbf{p}}_i = \dot{l}_{i-1}R_i \{ -1 \mathbf{p}_{i-1} + l_{i-1} \mathbf{\omega}_{i-1} \times \mathbf{p}_{i-1} \} \]  

\[ i^\dot{\mathbf{s}}_i = l_{i-1} \mathbf{\omega}_i \times \dot{\mathbf{s}}_i + \dot{l}_{i-1} \mathbf{\omega}_i \times (\mathbf{\omega}_i \times \dot{\mathbf{s}}_i) \]  

Table 1. Physical parameters.

<table>
<thead>
<tr>
<th>Link</th>
<th>( l_i )</th>
<th>( m_i )</th>
<th>( d_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Head</td>
<td>0.24</td>
<td>4.5</td>
<td>0.5</td>
</tr>
<tr>
<td>Upper body</td>
<td>0.41</td>
<td>21.5</td>
<td>10.0</td>
</tr>
<tr>
<td>Middle body</td>
<td>0.1</td>
<td>2.0</td>
<td>10.0</td>
</tr>
<tr>
<td>Lower body</td>
<td>0.1</td>
<td>2.0</td>
<td>10.0</td>
</tr>
<tr>
<td>Upper arm</td>
<td>0.31</td>
<td>2.3</td>
<td>0.03</td>
</tr>
<tr>
<td>Lower arm</td>
<td>0.24</td>
<td>1.4</td>
<td>1.0</td>
</tr>
<tr>
<td>Hand</td>
<td>0.18</td>
<td>0.4</td>
<td>2.0</td>
</tr>
<tr>
<td>Waist</td>
<td>0.27</td>
<td>2.0</td>
<td>10.0</td>
</tr>
<tr>
<td>Upper leg</td>
<td>0.38</td>
<td>7.3</td>
<td>10.0</td>
</tr>
<tr>
<td>Lower leg</td>
<td>0.40</td>
<td>3.4</td>
<td>10.0</td>
</tr>
<tr>
<td>Foot</td>
<td>0.07</td>
<td>1.3</td>
<td>10.0</td>
</tr>
</tbody>
</table>

| Total weight [kg] | 64.2 | — |
| Total height [m]  | 1.7  | — |

Here, \( i^\ddot{\mathbf{p}}_i \) represents position vector from the origin of \((i-1)\)-th link to the one of i-th, \( i^\dot{\mathbf{s}}_i \) is defined as gravity center position of i-th link and \( e_{ci} \) is unit vector that shows rotational axis of i-th link. However, velocity and acceleration of 4th link transmit to 8th link and ones of 1oth link transmit to 11th, 14th and 17th link directly because of ramification mechanisms.

2.2. Backward Inverse Dynamical Calculations

After the above forward kinetic calculation has been done, contrarily inverse dynamical calculation from top to base link are shown as follow. Newton equation and Euler equation of i-th link are represented by Eqs. (9), (10) when \( \mathbf{I}_i \) is defined as inertia tensor of i-th link. Here, \( \mathbf{f}_i \) and \( \mathbf{m}_i \) in \( \Sigma_j \) show the force and moment exerted on i-th link from \((i+1)\)-th link.

\[ i^\mathbf{f}_i = (R_{i+1})^T_i f_{i+1} + m_i i^\mathbf{s}_i \]  

\[ i^\mathbf{f}_i = \dot{R}_{i+1}^T f_{i+1} + m_i i^\mathbf{s}_i \]  

\[ i^\mathbf{f}_i = \dot{R}_{i+1}^T f_{i+1} + m_i i^\mathbf{s}_i \]  

\[ i^\mathbf{f}_i = \dot{R}_{i+1}^T f_{i+1} + m_i i^\mathbf{s}_i \]  

Fig. 1. Definition of humanoid’s link, joint and whole body.
Equation (15), and the nonlinear force of the prismatic joint, the torque onto the 1-st link can be made by

\[ \tau = \begin{bmatrix} \frac{\partial C_1}{\partial q} & \frac{\partial C_2}{\partial q} & \frac{\partial C_3}{\partial q} \end{bmatrix} \begin{bmatrix} \dot{q}_1 + \dot{q}_2 \tau_1 + \dot{q}_3 f_{\text{cy}} \end{bmatrix} \]  

Similarly, force and torque of 11th, 14th and 17th links transmit to 10th link directly. Then, rotational motion equation of i-th link is obtained as Eq. (13) by making inner product of induced torque onto the i-th link’s unit vector \( e_i \) around rotational axis:

\[ \tau_i = e_i^T \tau_i + d_i \dot{q}_i. \]  

However, when the supporting-leg (1-st link) is slipping (prismatic joint), the torque onto the 1-st link can be calculated by following equation:

\[ f_1 = e_1^T f_1 + \mu_k \gamma_0. \]  

Finally, we get motion equation with one leg standing as:

\[ M(q) \ddot{q} + h(q, \dot{q}) + g(q) + D\dot{q} = \tau, \]  

where \( \tau = [f_1, \tau_1, \tau_2, \ldots, \tau_{17}] \) is input torque, \( M(q) \) is inertia matrix, both of \( h(q, \dot{q}) \) and \( g(q) \) are vectors which indicate Coriolis force, centrifugal force and gravity. When the supporting-leg is slipping, the \( D = \text{diag}[\mu_k, d_1, d_2, \ldots, d_{17}] \) is a matrix which means coefficients of joints and between foot and ground. And \( q = [\gamma_0, q_1, q_2, \ldots, q_{17}] \) means the angles of joints and the relative position between foot and ground. When the supporting-leg is slipping, the variable vector \( q \) consists of \( q = [\gamma_0, q_1, q_2, \ldots, q_{17}] \). The viscous friction of y-axis (slipping axis) can be described as \( \mu_k \gamma_0 \) that is included in left-side of Eq. (15), and the nonlinear force generated by reaction force to the supporting-leg \( f_0 \) is made by \( f_0 = \mu_k \gamma_0 \) where \( f_0 \) is normal force exerting to supporting-leg caused by dynamical coupling of the humanoid body given by Eqs. (9) and (10), and \( \mu_k \) is dynamical friction coefficient.

### 2.3. Constraint Conditions for Free-Leg Model

Making Free-leg contact with ground, free-leg appears with the position \( \gamma_0 \) or angle \( q_t \) to the ground being constrained. Also, when free-leg’s velocity in traveling direction \( \dot{y}_0 \) is less than 0.01[m/s], the free-leg will be constrained in acceleration by the static friction. The constraints of foot’s z-axis position, heel’s rotation and foot’s y-axis position can be defined as \( C_1, C_2 \) and \( C_3 \) respectively, these constraints can be written as follow, where \( r(q) \) means the free-leg’s heel or toe position in \( \Sigma_W \).

\[ C(r(q)) = \begin{bmatrix} C_1(r(q)) \\ C_2(r(q)) \\ C_3(r(q)) \end{bmatrix} \]  

Then, robot’s equation of motion with external force \( f_{\text{nc}}, \) friction force \( f_1, \) external torque \( \tau_n \) and external force \( f_{\text{cy}} \) corresponding to \( C_1, C_2 \) and \( C_3 \) can be derived as:

\[ M(q) \ddot{q} + h(q, \dot{q}) + g(q) + D\dot{q} = \tau + j^T_{\text{cy}} f_{\text{nc}} - j^T_1 f_1 + j^T_1 \tau_1 + j^T_{\text{cy}} f_{\text{cy}} \]  

where \( j_{\text{cy}}, j_1, j_2, \) and \( j_{\text{cy}} \) are defined as:

\[ j^T_{\text{cy}} = \left( \frac{\partial C_1}{\partial q} \right)^T \left( \frac{1}{\left\| \dot{q} \right\|} \right), \quad j^T_1 = \left( \frac{\partial r}{\partial q} \right)^T \frac{1}{\left\| \dot{q} \right\|}, \quad j^T_{\text{cy}} = \left( \frac{\partial C_1}{\partial q} \right)^T \frac{1}{\left\| \dot{q} \right\|} \]  

It is common sense that (i) \( f_{\text{nc}} \) and \( f_1 \) are orthogonal, and (ii) value of \( f_1 \) is decided by \( f_1 = K f_{\text{nc}} \) (0 < \( K \leq 1 \)). The differentiating Eq. (16) by time for two times, we can derive the constraint condition of \( \dot{q} \).

\[ \begin{bmatrix} \left( \frac{\partial C_1}{\partial q} \right)^T \dot{\dot{q}} + \left( \frac{\partial C_1}{\partial q} \right)^T \left( \frac{\partial C_1}{\partial q} \right) \dot{q} \end{bmatrix} = 0. \]  

(19) \((i = 1, 2)\)

Making the \( \dot{q} \) in Eqs. (17) and (20) be identical, we can obtain the equation of contacting motion as follow.

\[ \begin{bmatrix} M(q) - j^T_{\text{cy}} f_{\text{nc}} & j^T_{\text{cy}} f_{\text{cy}} \end{bmatrix} \begin{bmatrix} \dot{q} \\ f_{\text{nc}} \end{bmatrix} = \begin{bmatrix} \tau - h(q, \dot{q}) - g(q) - D\dot{q} \\ -q^T \left( \frac{\partial C_1}{\partial q} \right) \end{bmatrix} \]  

(20)

### 2.4. Calculation of Bumping

When swing-leg attaches to ground, we need to consider bumping motion. There are two kinds of bumping concerning heel and toe. We denote dynamics of bumping between the heel and the ground below. By integrating Eq. (17) under \( \tau_n = 0 \) in time, equation of striking moment can be obtained as follows.

\[ M(q) \ddot{q}(t_{i+}) = M(q) \dot{q}(t_{i+}) + (j^T_{\text{cy}} - j^T_1 K) F_{\text{im}}. \]  

Eq. (21) describes the bumping in z-axis of \( \Sigma_W \) between the heel and the ground. \( \dot{q}(t_{i+}) \) and \( \dot{q}(t_{i+}) \) are angular velocity after and before the strike respectively.

\[ F_{\text{im}} = \lim_{t_i \to t_{i+}} \int_{t_i}^{t_{i+}} f_t dt. \]  

(22)
Then, the equation of matrix formation in the case of heel’s bumping can be obtained as follows.

$$\begin{bmatrix}
M(q) & \left(f_i^e - f_i^j - j_i^j K\right) \\
\frac{\partial C_1}{\partial q} & 0
\end{bmatrix}
\begin{bmatrix}
\dot{q}(t_1^+) \\
0
\end{bmatrix}
= \begin{bmatrix}
M(q) \dot{q}(t_1^+) \\
0
\end{bmatrix}.$$  

(24)

We can derive the dynamics regarding the toe’s bumping based on the similar above process.

3. Validation of Model

3.1. Verification by Mechanical Energy

To verify this complex model, we use the mechanical energy conservation law. Because to verify the conservation of mechanical energy, the equation of motion must be correct. We make the model to do a free fall with the input torque $\tau = 0$ and the viscous friction $D_i = 0$. In this case, there is no friction. So, it will has no discharge of energy during free fall. During the motion the mechanical energy will be saved at the initial potential energy. To derive the mechanical energy, it is necessary to calculate all of the potential energy, rotational energy and translational energy.

3.2. Calculation of Mechanical Energy

It is necessary to calculate the height of the center of gravity of each link before the calculation of the potential energy. We use the homogeneous transformation matrix to calculate it as following equation.

$$w z_{Gi} = w z_i + \frac{w z_{i+1} - w z_i}{2}.$$  

(25)

Here, $w z_{Gi}$ means the height of C.o.G of i-th link in world coordinate system $w z_i$ is the height of the joint which seen from the world coordinate. So, we can calculate the potential energy as following equation.

$$E_p = \sum_{i=1}^{17} m_i w z_i.$$  

(26)

Here, $E_p$ is the potential energy of the model. $m_i$ is the mass of each link. $g$ is the gravitational acceleration. Then, we can calculate the rotational energy as following equation.

$$E_k = \frac{1}{2} \sum_{i=1}^{17} \omega_i^T w I_i w \omega_i.$$  

(27)

To verify this model and check the effect of the friction on the slip motion. The simulation environment is set as shown in Fig. 2.

The initial conditions of experiment are shown as follow. The joint angle of the whole body are set to $q = 0$, and there are no viscous friction ($D = 0$) and input torque ($\tau = 0$). And the supporting-leg is constrained to ground with a surface-contact. So, the body of humanoid robot will fall freely, and the supporting-leg of Humanoid robot will have a slip motion on the ground. Because the coefficient of friction of the left side of the ground (darker part in Fig. 2) is set to $\mu_k = 0.3$, and the friction coefficient of the right side is set to $\mu_k = 0$, the total mechanical energy will be maintained when the supporting-leg moves on the surface of $\mu_k = 0$ and decrease monotonously when it does on the surface of $\mu_k = 0.3$.

The configuration in Fig. 2(a) was detected at the time designated by (a) in Fig. 3, and also (b) and (c) are the shapes at (b) and (c). Since the equation of mo-
motion used in this simulation represents the motion that the supporting-leg is kept to be contacting to the floor surface contact and the free-leg is not constraint, the free-leg can descend down below the ground surface as shown in Figs. 2(b) and (c).

Figure 3 shows the result of the mechanical energy and the changes of the coefficient of friction when the supporting-leg is in a slip motion. From Fig. 3, when the supporting-leg is staying at the right part of ground ($\mu_k = 0$), the total of mechanical energy is saved. But when it is staying at the right part ($\mu_k = 0.3$), the mechanical energy is discharged by the effect of friction. So, it can be seen that supporting-leg slip model is feasible.

3.3.2. Simulation Including Stick-Slip

Another simulation with nonlinear friction between floor and foot has been prepared to examine a stick-slip motion that can verify the humanoid model further. The experiment conditions are shown as follows. The state (stick or slip) of supporting-leg that is dominated by the stick-slip conditions are shown in Fig. 4. When the driving force exerting to supporting-leg from dynamical coupling of humanoid nonlinear model $f_{y0}$ is larger than the maximum static frictional force $f_{s0}$, the supporting-leg starts to slip. Here $f_{y0}$ means the normal force exerting to the foot, and when the slip velocity of supporting-leg $|\dot{y}_0|$ is less than $\epsilon$ (a very small value $\epsilon = 0.001$ m/s = 1 mm/s in this paper), the supporting-leg enters a stick state. During the supporting-leg in stick state, the coefficient of friction is set to $\mu_s = 0.8$, and when sticking the $\mu_k$ is equal to zero. And the body of humanoid robot will fall freely without any viscous friction ($D = 0$) and input torque ($\tau = 0$).

$$E_{\text{discharge}} = \int_0^\tau \mu_k |\dot{y}_0|^2 dt$$

Equation (30)

Figure 5 shows time profile of mechanical energy of humanoid’s free-fall motion including the stick-slip motion, and Fig. 6 shows the discharged energy caused by friction on floor, the calculation of discharged energy is shown in Eq. (30). Here, $\mu_k$ means the coefficient of friction when slipping, and when sticking the $E_{\text{discharge}}$ in Eq. (30) equal to zero since $|\dot{y}_0|$ is zero. In Figs. 5 and 6, when the supporting-leg is in the state of stick, the total of mechanical energy is remained unchanged. And the mechanical energy discharges while the the supporting-leg is slipping, and the value of discharged energy in Fig. 6 is consistent with the result in Fig. 5.

Furthermore the velocity and the y-axis position of supporting-leg are shown in Figs. 7 and 8. Fig. 7 shows that when the supporting-leg is in the state of stick, the velocity of slip equals to zero, and Fig. 8 shows that the y-axis position is also not changed in time. And the discharge of energy depends on the velocity of slip. When the supporting-leg is slipping fast, the discharge of energy also gets a higher rate. Conversely, when supporting-leg is slipping very slow or stopping, the discharge of energy is also small or kept unchanged. So, from this simulation, it also can be seen that supporting-leg slip-stick model is feasible.
Fig. 8. y-position of supporting-leg.

Fig. 9. Configurations during free-fall simulation shown in Fig. 8. (a) the configuration at time A designated in Fig. 8, and (b) and (c) corresponds to time B and C.

Figure 9 shows their shapes of the humanoid while the simulated motion proceeds as shown in Figs. 5–8. The configuration in Fig. 9(a) was detected at the time designated by A in Figs. 7 and 8, and also (b) and (c) are the shapes at B and C in the both figures.

4. Gait of Walking Model

4.1. Transition of Gait

In this research, humanoid walking with the transition of gaits state as shown in Fig. 10. And the route of state transition is determined by the walking motion of humanoid. In other words, it depends on the solution of the dynamics in Eq. (20). The dynamics and the state variable which depends on the gaits has been selected. And it will transit to the next state when the branching condition is satisfied. All of the walking dynamical models as in Fig. 10 include the constrain conditions as shown in Table 2.

4.2. Switching Gaits

When the humanoid robot is walking, the gaits need to continue seamlessly by the conditions written at the upside position or downside of the arrows in Fig. 10. For example, in a simulation of common walking (route-2),
the initial condition is set to state 2 shown in Fig. 10. When the free-leg touch the floor, meaning $z_{h} \leq 0$, the equation of motion is switched to state 6, and the free-leg start slipping. When the slipping stop, $|\dot{y}| < \varepsilon$, the state of motion is switched to state 10. Then the gait state will be switched continually to state 18, 9', 1 and goes back to state 2 by their switching conditions. The state number in rectangular box in Fig. 10 corresponds to the number of states in Table 2.

In this paper, gaits in Fig. 11 are verified by using the mechanical energy conservation law as the same in Section 3. Two verification experiments about the gaits switching have been prepared. The switching route-1,2 in Fig. 10 is used. The route-1 actually shows a common pattern to fall to the ground. And the route-2 shows the common bipedal walking. In the route-1 and 2, the viscous friction coefficient of each joints and the friction coefficient the foot and the ground are set to zero. So the transition of state in route-1 and 2 are transferred as shown in Fig. 11. Because the gait number (6) in the route-2 occur in a very short moment after the foot contact to the ground, and it switch to the gait (10) immediately. So, in the route-2, the gait (6) not discussed in this experiment.

Figure 12 shows the shapes of humanoid during the switching simulation of route-1. From Fig. 12, the initial state of gaits is (4). When the heel and toe of free-leg bumping with the floor, the state is switched to number (8) and (16) continually.

### Table 2. Possible states for humanoid’s walking, where the state number from (1) to (20) corresponds to the state number in rectangular in Fig. 10.

<table>
<thead>
<tr>
<th>States in Fig. 10</th>
<th>State variables and constraining force and torque (Lagrange Multiplier)</th>
<th>Constraints (reference)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) $q = [q_{1}, q_{2}, \cdots, q_{17}]^{T}$</td>
<td>$C_{bc} = 0$</td>
<td></td>
</tr>
<tr>
<td>(2) $q = [q_{1}, q_{2}, \cdots, q_{17}]^{T}$</td>
<td>$C_{bc} = 0$</td>
<td></td>
</tr>
<tr>
<td>(3) $q = [q_{1}, q_{2}, \cdots, q_{17}]^{T}$</td>
<td>$C_{bc} = 0$</td>
<td></td>
</tr>
<tr>
<td>(4) $q = [q_{1}, q_{2}, \cdots, q_{17}]^{T}$</td>
<td>$C_{bc} = 0$</td>
<td></td>
</tr>
<tr>
<td>(5) $q = [q_{1}, q_{2}, \cdots, q_{17}]^{T}$</td>
<td>$C_{bc}, C_{h} = 0$</td>
<td></td>
</tr>
<tr>
<td>(6) $q = [q_{1}, q_{2}, \cdots, q_{17}]^{T}$</td>
<td>$C_{bc}, C_{h} = 0$</td>
<td></td>
</tr>
<tr>
<td>(7) $q = [q_{1}, q_{2}, \cdots, q_{17}]^{T}$</td>
<td>$C_{bc}, C_{h} = 0$</td>
<td></td>
</tr>
<tr>
<td>(8) $q = [q_{1}, q_{2}, \cdots, q_{17}]^{T}$</td>
<td>$C_{bc}, C_{h} = 0$</td>
<td></td>
</tr>
<tr>
<td>(9) $q = [q_{1}, q_{2}, \cdots, q_{17}]^{T}$</td>
<td>$C_{bc}, C_{h} = 0$</td>
<td></td>
</tr>
<tr>
<td>(10) $q = [q_{1}, q_{2}, \cdots, q_{17}]^{T}$</td>
<td>$C_{bc}, C_{h} = 0$</td>
<td></td>
</tr>
<tr>
<td>(11) $q = [q_{1}, q_{2}, \cdots, q_{17}]^{T}$</td>
<td>$C_{bc}, C_{h} = 0$</td>
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<tr>
<td>(12) $q = [q_{1}, q_{2}, \cdots, q_{17}]^{T}$</td>
<td>$C_{bc}, C_{h} = 0$</td>
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<td>(13) $q = [q_{1}, q_{2}, \cdots, q_{17}]^{T}$</td>
<td>$C_{bc}, C_{h} = 0$</td>
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<td>(14) $q = [q_{1}, q_{2}, \cdots, q_{17}]^{T}$</td>
<td>$C_{bc}, C_{h} = 0$</td>
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<td>(15) $q = [q_{1}, q_{2}, \cdots, q_{17}]^{T}$</td>
<td>$C_{bc}, C_{h} = 0$</td>
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<td>(16) $q = [q_{1}, q_{2}, \cdots, q_{17}]^{T}$</td>
<td>$C_{bc}, C_{h} = 0$</td>
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</tr>
<tr>
<td>(17) $q = [q_{1}, q_{2}, \cdots, q_{17}]^{T}$</td>
<td>$C_{bc}, C_{h} = 0$</td>
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</tr>
<tr>
<td>(18) $q = [q_{1}, q_{2}, \cdots, q_{17}]^{T}$</td>
<td>$C_{bc}, C_{h} = 0$</td>
<td></td>
</tr>
<tr>
<td>(19) $q = [q_{1}, q_{2}, \cdots, q_{17}]^{T}$</td>
<td>$C_{bc}, C_{h} = 0$</td>
<td></td>
</tr>
<tr>
<td>(20) $q = [q_{1}, q_{2}, \cdots, q_{17}]^{T}$</td>
<td>$C_{bc}, C_{h} = 0$</td>
<td></td>
</tr>
</tbody>
</table>

The result about the mechanical energy conservation law of the route-1 and 2 are shown in Figs. 13 and 14. From these two figures, when the foot is contact with the ground, the mechanical energy has been dissipated by bumping, but in each gait, the mechanical energy has been kept. So, considering that the mechanical energy conservation law has been satisfied in each gait, the gait switching models are verified to be noncontradiction.
5. Conclusion

In this paper, we have proposed a walking model of humanoid including slipping, bumping, surface-contacting and point-contacting of foot, whose dynamical equation is derived by Newton-Euler method with constraint condition.

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