A Proposal of Temperature Control Model for Two Dimensional Aluminum Plate Using Two Degree-of-Freedom Generalized Predictive Control

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Abstract: This paper considers an application of two degree-of-freedom generalized predictive control (Two DOF GPC) to the model of temperature control experimental device of two dimensional aluminum plate. In our research, Two DOF GPC can achieve to design the output response for the aluminum plate model with modeling error or disturbance, and without them independently. The present study aims at three dimensional temperature control by expanding the model of the one dimensional aluminum plate into two dimensional one.

Keywords: Two degree-of-freedom, Generalized predictive control, Temperature control

1. INTRODUCTION

The present study aims at three dimensional temperature control by expanding the model of the one dimensional aluminum plate into two dimensional. GPC technique has been first proposed by Clarke and others in 1987 [1]. The control method has features that the objective function includes prediction and control horizons, and control signals are computed by receding their horizons. With these features, the control strategy has been accepted by many of practical engineers and applied widely in industry [2]. Although our method can achieve to design the output response for the aluminum plate model with modeling error or disturbance, and without them independently, the previous model was one dimensional model [3][4][5].

There are a lot of studies about temperature control. For example, the temperature control has been studied by Zhang and others in 2002 was employed GPC as a method, but the control itself deals with PID control by the self-tuning based GPC.[7] Further, the temperature control for the purpose of temperature uniformity by Nanno and others in 2002 has studies gradient of temperature control method by using the PID control.[8]

Our study by using Two DOF GPC about temperature control is unique in other one. We consider the application to industrial fields by using this method. But, It cannot be said to be sufficient for the application to industry. So this paper aims at three dimensional temperature control by expanding the model of the one dimensional aluminum plate into two dimensional one in order to develop thermotherapy machine and to produce products which are made from thermoplastic materials.

Therefore, this paper firstly constructs two dimensional aluminum plate model for temperature control by extending the conventional model. Moreover, the simulation result with temperature control of the derived model by using two DOF GPC is shown.

2. DERIVATION OF CONTROLLER

Consider the following m-input m-output system:

\[
\begin{align*}
x[k+1] &= Ax[k] + Bu[k] \\
y[k] &= Cx[k]
\end{align*}
\]

where \(x[k], y[k], u[k]\) and \(k\) denote state variable, output, input and time step. \(A, B, C\) are constant matrix. The steady state values \(x_\infty, u_\infty\) of \(x[k]\) and \(y_\infty\) of \(y[k]\) are derived as follows.

\[
\begin{align*}
x_\infty &= Ax_\infty + Bu_\infty \\
y_\infty &= Cx_\infty
\end{align*}
\]

Subtracting Eq.(3) and (4) from Eq.(1) and (2).

\[
\begin{align*}
x[k+1] - x_\infty &= A(x[k] - x_\infty) + B(u[k] - u_\infty) \\
y[k] - y_\infty &= C(x[k] - x_\infty)
\end{align*}
\]

It is possible to provide the following deviation system by defining \(\bar{x}[k] = x[k] - x_\infty\), \(\bar{y}[k] = y[k] - y_\infty\) and \(\bar{u}[k] = u[k] - u_\infty\).

\[
\begin{align*}
\bar{x}[k+1] &= A\bar{x}[k] + B\bar{u}[k] \\
\bar{y}[k] &= C\bar{x}[k]
\end{align*}
\]

For this deviation system, the output of \(j\)-steps ahead \(\bar{y}[k+j]\) can be calculated as follows.

\[
\bar{y}[k+j] = CA^j\bar{x}[k] + \sum_{i=1}^{j} CA^{i-1}B\bar{u}[k+j-i]
\]

Since it is assumed that there is no disturbance here, the predicted value of the output of \(j\)-steps ahead, \(\hat{y}[k+j]\), is equal to Eq.(7). Further, to express a vector form \(\hat{y}[k+j]\), the following vectors and matrices are defined.

\[
\hat{y}[k] = [\bar{y}^T[k + N_1][k] \cdots \bar{y}^T[k + N_2][k]]^T
\]
\[
\hat{u}[k] = [\bar{u}^T[k] \cdots \bar{u}^T[k + N_u - 1]]^T
\]
3. CONSTRUCTION OF TWO DOF GPC

The following control law is considered by including integral compensation in order to construct two DOF GPC.

\[ u[k] = F_0 x[k] + H_0 r + G_0 z[k] \]  \hspace{1cm} (14)

Where \( z[k] \) is integral compensation, \( G_0 \) is integral gain. The control law (Eq.(14)) can make the steady state error zero. However without disturbance and modeling error, it may lead to increase in the control input and delay of tracking the target value in the transient response. So we make the effect of integral compensation appear only when the disturbance and modeling error exist. Also as \( e[k] = r - y[k] \), which is the difference between the target value and the output, is used for integral compensation as follows.

\[ w[k] = \frac{1}{\Delta} e[k] \hspace{1cm} (\Delta = 1 - z^{-1}) \]  \hspace{1cm} (15)

In the case that the disturbance or modeling error does not exist, Eq.(14) should be equal to Eq.(13), in order to derive two DOF GPC law. Therefore the following equation is considered by substituting Eq.(13) into Eq.(1), and subtracting \( x[k] \) from both sides.

\[ x[k+1] - x[k] = (A - I + BF_0)x[k] + BH_0 r \]

Then

\[ x[k] = (A - I + BF_0)^{-1} x[k+1] \]
\[ - (A - I + BF_0)^{-1} x[k] \]
\[ -(A - I + BF_0)^{-1} BH_0 r \]  \hspace{1cm} (16)

By substituting the above equation into \( e[k] \) and using Eq.(12), we have.

\[ e[k] = -C(A - I + BF_0)^{-1} (x[k+1] - x[k]) \]

That is, if the disturbance and modeling error do not exist, the integral compensation by the tracking error can be calculated as follows.

\[ w'[k] = \frac{1}{\Delta} e[k] \]
\[ = -C(A - I + BF_0)^{-1} (x[k+1] - x[k]) \]
\[ - (x[k] - x[0]) \]  \hspace{1cm} (17)

Where \( w'[0] \) is assumed to be 0. Since Eq.(17) represents the integral compensation when the disturbance does not exist, the integral compensation \( z[k] \) of two DOF GPC is given by the following equation.

\[ z[k] = w'[k] - w''[k] \]

From this equation, it always becomes \( z[k] = 0 \) if the disturbance and modeling error does not exist, that is, the effect of the integral compensation does not appear. Thus, the control law of two DOF GPC is given.
4. SIMULATION

4.1. Derivation of Model

Firstly the following model shown in Fig.1 is considered.

\[ u \]
\[ y \]
Fig. 1 Model of aluminum plate

\[ w \]
\[ q \]

Where \( w \) is heat flow ratio \([W/m^2]\), \( q \) is heat transfer coefficient \([W/m^2K]\), \( h \) is heat transfer coefficient \([W/m^2K]\).

The law of heat conduction is given by, 

\[ dQ = mc \cdot d\theta \]  \hspace{1cm} (21)

\( c \) is specific heat \([J/kgK]\), \( m \) is mass of each part \([kg]\).

Then the following equations of the system are obtained from the state variables of Eq.(18) and the laws of Eq.(19),(20) and (21).

\[
mc \frac{dx_1}{dt} = -(k(\frac{1}{3}S_1 + \frac{4}{9}S_3) + \lambda_f (S_1 \frac{S_1}{d_1} + \frac{2S_2}{d_2}))x_1 + (\lambda_f S_1 \frac{S_1}{d_1})x_4 + (\lambda_f S_2 \frac{S_2}{d_2})x_4 + (\lambda_f S_1 \frac{S_1}{d_1})x_4
\]

\[
mc \frac{dx_2}{dt} = -(k(\frac{1}{3}S_1 + \frac{2}{9}S_1) + \lambda_f (S_1 \frac{S_1}{d_1} + \frac{2S_2}{d_2}))x_2 + (\lambda_f S_1 \frac{S_1}{d_1})x_4 + (\lambda_f S_2 \frac{S_2}{d_2})x_4 + (\lambda_f S_1 \frac{S_1}{d_1})x_4
\]

\[
mc \frac{dx_3}{dt} = -(k(\frac{1}{3}S_1 + \frac{3}{9}S_2) + \lambda_f (S_1 \frac{S_1}{d_1} + \frac{2S_2}{d_2}))x_3 + (\lambda_f S_1 \frac{S_1}{d_1})x_6 + (\lambda_f S_2 \frac{S_2}{d_2})x_6 + (\lambda_f S_1 \frac{S_1}{d_1})x_6
\]

\[
mc \frac{dx_4}{dt} = -(k(\frac{1}{3}S_1 + \frac{4}{9}S_3) + \lambda_f (S_1 \frac{S_1}{d_1} + \frac{2S_2}{d_2}))x_4 + (\lambda_f S_1 \frac{S_1}{d_1})x_8 + (\lambda_f S_2 \frac{S_2}{d_2})x_8 + (\lambda_f S_1 \frac{S_1}{d_1})x_8
\]

\[
mc \frac{dx_5}{dt} = -(k(\frac{2}{9}S_1 + \lambda_f (\frac{S_1}{d_1} + \frac{2S_2}{d_2}))x_5 + u_1 + (\lambda_f S_1 \frac{S_1}{d_1})x_4 + (\lambda_f S_2 \frac{S_2}{d_2})x_4 + (\lambda_f S_1 \frac{S_1}{d_1})x_4
\]

\[
mc \frac{dx_6}{dt} = -(k(\frac{1}{3}S_1 + \frac{4}{9}S_1) + \lambda_f (2S_1 \frac{S_1}{d_1} + \frac{2S_2}{d_2}))x_6 + (\lambda_f S_1 \frac{S_1}{d_1})x_8 + (\lambda_f S_2 \frac{S_2}{d_2})x_8 + (\lambda_f S_1 \frac{S_1}{d_1})x_8
\]

\[
mc \frac{dx_7}{dt} = -(k(\frac{4}{9}S_3 + \lambda_f (\frac{S_1}{d_1} + \frac{2S_2}{d_2}))x_7 + u_1 + (\lambda_f S_1 \frac{S_1}{d_1})x_8 + (\lambda_f S_2 \frac{S_2}{d_2})x_8 + (\lambda_f S_1 \frac{S_1}{d_1})x_8
\]

\[
mc \frac{dx_8}{dt} = -(k(\frac{1}{3}S_1 + \frac{2}{9}S_1) + \lambda_f (\frac{S_1}{d_1} + \frac{2S_2}{d_2}))x_8 + (\lambda_f S_1 \frac{S_1}{d_1})x_9 + (\lambda_f S_2 \frac{S_2}{d_2})x_9 + (\lambda_f S_1 \frac{S_1}{d_1})x_9
\]

The parameters of the aluminum plate thermal process model are given in Table.1

| Density of aluminum: \( \rho = 2700[kg/m^3] \) |
| Specific Heat: \( c = 0.917[kJ/kgK] \) |
| Heat transfer coefficient: \( k = 20[W/m^2K] \) |
| Thermal conductivity: \( \lambda_f = 238[W/mK] \) |
| Height and Length: \( l = 250[mm] \) |
| Width: \( w = 10[mm] \) |

Table 1 Parameters for the model

The state variables are defined as the following.

\[ x_n = T_n - T_o \]  \hspace{1cm} (18)

This model is nine dimensional model with dividing into 3 \times 3 lengthwise and crosswise and \( n = 1, 2, \ldots, 9 \) to the right from the upper left. Where \( T_n \) is temperature of each part of the aluminum plate, \( T_o \) is ambient temperature. Then, three laws are used in the derivation of the model.

Fourier’s law of heat conduction is given by,

\[ q = -\lambda_f \frac{d\theta}{dn} \]  \hspace{1cm} (19)

Where \( q \) is heat flow ratio \([W/m^2]\), \( \lambda_f \) is thermal conductivity \([W/mK]\), \( d\theta/dn \) is temperature gradient \([K/m]\).

Newton’s law of cooling is given by,

\[ q = h(\theta_s - \theta_f) \]  \hspace{1cm} (20)
\[
\frac{mc}{dt} \frac{dx}{dt} = -(k(\frac{1}{3}S_1 + \frac{1}{3}S_2 + \frac{2}{9}S_3) + \lambda f(S_1d_1 + S_2d_2))x_9 + (\lambda fS_1d_1)x_6 + (\lambda fS_2d_2)x_8
\]

From these equations, the state space equations were obtained. For the sake of simple expression of the coefficients for the above equations, each coefficient of the equation for \(x_i, i = 1, \ldots, 9\) is renamed as \(a_{ij}, i = 1, \ldots, 9, j = 1, \ldots, 9\).

\[
\begin{align*}
mc \frac{dx_1}{dt} &= a_{11}x_1 + a_{12}x_2 + a_{14}x_4 \\
mc \frac{dx_2}{dt} &= a_{22}x_2 + a_{21}x_1 + a_{23}x_3 + a_{25}x_5 \\
mc \frac{dx_3}{dt} &= a_{33}x_3 + a_{32}x_2 + a_{36}x_6 \\
mc \frac{dx_4}{dt} &= a_{44}x_4 + a_{41}x_1 + a_{45}x_5 + a_{47}x_7 \\
mc \frac{dx_5}{dt} &= a_{55}x_5 + a_{52}x_2 + a_{54}x_4 + a_{56}x_6 + a_{58}x_8 + u_1 \\
mc \frac{dx_6}{dt} &= a_{66}x_6 + a_{63}x_3 + a_{65}x_5 + a_{69}x_9 \\
mc \frac{dx_7}{dt} &= a_{77}x_7 + a_{74}x_4 + a_{78}x_8 \\
mc \frac{dx_8}{dt} &= a_{88}x_8 + a_{85}x_5 + a_{87}x_7 + a_{89}x_9 \\
mc \frac{dx_9}{dt} &= a_{99}x_9 + a_{96}x_6 + a_{98}x_8
\end{align*}
\]

The result of the above state space equations is given.

\[
x(k + 1) = Ax(k) + Bu(k) \\
y(k) = Cx(k)
\]

The system matrices \(A, B\) and \(C\) are defined as follows,

\[
A = \frac{1}{mc}
\begin{bmatrix}
    a_{11} & a_{12} & 0 & a_{14} & 0 & 0 & 0 & 0 & 0 \\
    a_{21} & a_{22} & a_{23} & 0 & a_{25} & 0 & 0 & 0 & 0 \\
    0 & a_{32} & a_{33} & 0 & 0 & 0 & 0 & 0 & 0 \\
    a_{41} & 0 & 0 & a_{44} & a_{45} & 0 & a_{47} & 0 & 0 \\
    a_{52} & 0 & a_{54} & a_{55} & a_{56} & 0 & a_{58} & 0 & 0 \\
    a_{53} & 0 & a_{56} & a_{66} & 0 & 0 & 0 & 0 & 0 \\
    0 & a_{74} & 0 & 0 & a_{77} & a_{78} & 0 & 0 & 0 \\
    a_{85} & 0 & a_{87} & a_{88} & a_{89} & 0 & 0 & 0 & 0 \\
    a_{96} & 0 & a_{98} & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

\[
B = \frac{1}{mc}
\begin{bmatrix}
    0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

\[
C =
\begin{bmatrix}
    x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 & x_8 & x_9
\end{bmatrix}
\]

Where the sampling time is 20[s]. Then the state space representation and the derivation of the aluminum plate temperature control model in the nine dimensional has been completed. It is noticed that although we assume that the obtained model is single input single output system and the positions of input and output are \(n = 5\) (located in the center part of the model), their positions can be changed easily.

### 4.2. Confirmation of the Model

In order to check the validity of the model, the step response for the model which is determined in section 4, is checked. In this case, it was confirmed spread of heat is weather it is natural. From Fig.7 to Fig.12, the temperature increase and their distributions are shown in the case that the heat input is applied at each part. At this time, the amount of heat input is always constant and are applying with 15[W]. Further, the output when the inputs to \(n_1, n_3, n_7\) and \(n_9\) is confirmed by Fig.7. When the inputs to \(n_2, n_4, n_6\) and \(n_8\) were given, the result is shown in Fig.9. Moreover, when the input to \(n_5\) was given, the result is shown in Fig.11. Also, Fig.8, 10 and 12 show the temperature distribution of each part (in the case for the input to \(n_1, n_2\) or \(n_5\)) at \(t = 500[s], 1000[s]\) and 4500[s].

![Fig. 3 The Simulation Model of Aluminum](image)

![Fig. 4 Temperature increase(input to \(n_1, n_3, n_7\) and \(n_9\))](image)

![Fig. 5 Temperature distribution (input to \(n_1\))](image)

![Fig. 6 Temperature increase(input to \(n_2, n_4, n_6\) and \(n_8\))](image)
It found that the heat spreads concentrically from the input point based on the results in Fig.7, 9 and 11 and the results in Fig.8, 10 and 12. Therefore, it can find that this model is appropriate.

4.3. Expansion in the Height Direction

In order to derive three dimensional model of aluminum plate thermal process system, based on the model of previous section, the extension to the vertical direction for the aluminum plate thermal process is considered in this section. Fig.13 shows the extended model and Fig.14, 15 and 16 give the results of temperature increase.

Although there is no temperature difference between $n_i$ and $n_{i+5}(i = 1, \cdots, 5)$, it seems to be caused by thinness of the aluminum plate model. But from Fig.14, 15 and 16, the temperature increase has a tendency that closer position is to the input position, the higher temperature appears. Therefore it can find that the extended model is also appropriate.

5. APPLICATION OF THE CONTROL LAW

On the model obtained in the previous section, the predictive control systems for the three cases are applied. Control target of temperature is to increase 4[K] from ambient temperature. In addition, the disturbance temperature 0.5[K] is added at $t = 5000[\text{s}]$. The design parameters of GPC are given in Table.2.
\[
N_1 = 10 \\
N_2 = 120 \\
N_u = 2 \\
\Lambda = 0.01I \\
G_0 = 0.2I \\
\]

Table 2 Parameters for GPC

6. CONCLUSION

This paper extended the aluminum plate temperature control experimental device model in two dimensional model from one dimensional model which is used in the previous study. The simulation was conducted in order to verify the validity of the model. That is, the figures for temperature increase and the color maps for temperature distribution are given to shown the relations between input and output. As future works, the model in three dimensions must be constructed and its simulation is investigated in order to confirm the validity of the model. Moreover the number of divisions of the model should increase because of aiming at accuracy of calculating result.

REFERENCES