SICE Annual Conference 2014
September 9-12, 2014, Hokkaido University, Sapporo, Japan

A Proposal of Temperature Control Model for Two Dimensional Aluminum Plate Using Two Degree-of-Freedom Generalized Predictive Control

Naoki Hosoya[1] , Akira Yanou[1] , Mamoru Minami[1] and Takayuki Matsuno[1]

[1]Graduate School of Natural Science and Technology, Okayama University, Okayama, Japan
(Tel: +81-86-251-8924; E-mail: en422871@s.okayama-u.ac.jp)

Abstract: This paper considers an application of two degree-of-freedom generalized predictive control (Two DOF GPC) to the model of temperature control experimental device of two dimensional aluminum plate. In our research, Two DOF GPC can achieve to design the output response for the aluminum plate model with modeling error or disturbance, and without them independently. The present study aims at three dimensional temperature control by expanding the model of the one dimensional aluminum plate into two dimensional one.

Keywords: Two degree-of-freedom, Generalized predictive control, Temperature control

1. INTRODUCTION

The present study aims at three dimensional temperature control by expanding the model of the one dimensional aluminum plate into two dimensional. GPC technique has been first proposed by Clarke and others in 1987 [1]. The control method has features that the objective function includes prediction and control horizons, and control signals are computed by receding their horizons. With these features, the control strategy has been accepted by many of practical engineers and applied widely in industry [2]. Although our method can achieve to design the output response for the aluminum plate model with modeling error or disturbance, and without them independently, the previous model was one dimensional model [3][4][5].

There are a lot of studies about temperature control. For example, the temperature control has been studied by Zhang and others in 2002 was employed GPC as a method, but the control itself deals with PID control by the self-tuning based GPC.[7]
Further, the temperature control for the purpose of temperature uniformity by Nanno and others in 2002 has studies gradient of temperature control method by using the PID control.[8]
Our study by using Two DOF GPC about temperature control is unique in other one. We consider the application to industrial fields by using this method.
But, It cannot be said to be sufficient for the application to industry. So this paper aims at three dimensional temperature control by expanding the model of the one dimensional aluminum plate into two dimensional one in order to develop thermotherapy machine and to produce products which are made from thermoplastic materials.

Therefore, this paper firstly constructs two dimensional aluminum plate model for temperature control by extending the conventional model. Moreover, the simulation result with temperature control of the derived model by using two DOF GPC is shown.

2. DERIVATION OF CONTROLLER

Consider the following m-input m-output system:

$$\mathbf{x}[k+1] = \mathbf{A}\mathbf{x}[k] + \mathbf{B}\mathbf{u}[k] \quad (1)$$

$$\mathbf{y}[k] = \mathbf{C}\mathbf{x}[k] \quad (2)$$

where $\mathbf{x}[k]$, $\mathbf{y}[k]$, $\mathbf{u}[k]$ and k denote state variable, output, input and time step. $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$ are constant matrix. The steady state values $\mathbf{x}_\infty$ of $\mathbf{x}[k]$, $\mathbf{u}_\infty$ of $\mathbf{u}[k]$ and $\mathbf{y}_\infty$ of $\mathbf{y}[k]$ are derived as follows.

$$\mathbf{x}_\infty = \mathbf{A}\,\mathbf{x}_\infty + \mathbf{B}\mathbf{u}_\infty \quad (3)$$

$$\mathbf{y}_\infty = \mathbf{C}\mathbf{x}_\infty \quad (4)$$

Subtracting Eq.(3) and (4) from Eq.(1) and (2).

$$\mathbf{x}[k+1] - \mathbf{x}_\infty = \mathbf{A}(\mathbf{x}[k] - \mathbf{x}_\infty) + \mathbf{B}(\mathbf{u}[k] - \mathbf{u}_\infty)$$

$$\mathbf{y}[k] - \mathbf{y}_\infty = \mathbf{C}(\mathbf{x}[k] - \mathbf{x}_\infty)$$

It is possible to provide the following deviation system by defining $\tilde{\mathbf{x}}[k] = \mathbf{x}[k] - \mathbf{x}_\infty$, $\tilde{\mathbf{y}}[k] = \mathbf{y}[k] - \mathbf{y}_\infty$ and $\tilde{\mathbf{u}}[k] = \mathbf{u}[k] - \mathbf{u}_\infty$.

$$\tilde{\mathbf{x}}[k+1] = \mathbf{A}\tilde{\mathbf{x}}[k] + \mathbf{B}\tilde{u}[k] \quad (5)$$

$$\tilde{\mathbf{y}}[k] = \mathbf{C}\tilde{\mathbf{x}}[k] \quad (6)$$

For this deviation system, the output of j-steps ahead $\tilde{\mathbf{y}}[k+j]$ can be calculated as follows.

$$\tilde{\mathbf{y}}[k+j] = \mathbf{C}\mathbf{A}^j\tilde{\mathbf{x}}[k] + \sum_{i=1}^{j}\mathbf{C}\mathbf{A}^{i-1}\mathbf{B}\tilde{\mathbf{u}}[k+j-i] \quad (7)$$

Since it is assumed that there is no disturbance here, the predicted value of the output of j-steps ahead, $\hat{\tilde{\mathbf{y}}}[k+j|k]$, is equal to Eq.(7). Further, to express a vector form $\hat{\tilde{\mathbf{y}}}[k+j|k]$, the following vectors and matrices are defined.

$$\hat{\tilde{\mathbf{Y}}}[k] = [\hat{\tilde{\mathbf{y}}}^T[k+N_1|k] \cdots \hat{\tilde{\mathbf{y}}}^T[k+N_2|k]]^T$$

$$\tilde{\mathbf{U}}[k] = [\tilde{\mathbf{u}}^T[k] \cdots \tilde{\mathbf{u}}^T[k+N_u-1]]^T$$

† Naoki Hosoya is the presenter of this paper.

$$\mathbf{H} = \begin{bmatrix} \mathbf{CA}^{N_1} \\ \mathbf{CA}^{N_1+1} \\ \vdots \\ \mathbf{CA}^{N_2} \end{bmatrix}$$

$$\mathbf{G} = \begin{bmatrix} \mathbf{CA}^{N_1-1}\mathbf{B} & \cdots & \mathbf{CB} & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \mathbf{CA}^{N_u-1}\mathbf{B} & \ddots & \ddots & \ddots & \mathbf{CB} \\ \vdots & & \vdots & & \vdots \\ \mathbf{CA}^{N_2-1}\mathbf{B} & \cdots & \cdots & \cdots & \mathbf{CA}^{N_2-N_u}\mathbf{B} \end{bmatrix}$$

Where $[N_1, N_2]$ and $[1, N_u]$ denote prediction horizon and control horizon. Then the output prediction $\hat{\tilde{\mathbf{Y}}}[k]$ can be expressed as follows.

$$\hat{\tilde{\mathbf{Y}}}[k] \quad = \quad \mathbf{H}\tilde{\mathbf{x}}[k] + \mathbf{G}\tilde{\mathbf{U}}[k] \tag{8}$$

To obtain a control law for realizing the target value tracking, the following performance index J is considered.

$$J \quad = \quad \tilde{\mathbf{Y}}^T[k]\tilde{\mathbf{Y}}[k] + \tilde{\mathbf{U}}^T[k]\mathbf{\Lambda}\tilde{\mathbf{U}}[k] \tag{9}$$

By substituting Eq.(8) into Eq.(9) and having partial differentiation for $\tilde{\mathbf{U}}[k]$, the control law is obtained.

$$\tilde{\mathbf{U}}[k] \quad = \quad -(\mathbf{G}^T\mathbf{G} + \mathbf{\Lambda})^{-1}\mathbf{G}^T\mathbf{H}\tilde{\mathbf{x}}[k]$$

Since $\mathbf{u}[k]$ is the first element of the $\tilde{\mathbf{U}}[k]$, the control input is given as follows.

$$\mathbf{u}[k] \quad = \quad \mathbf{F}_0\mathbf{x}[k] - \mathbf{F}_0\mathbf{x}_\infty + \mathbf{u}_\infty \tag{10}$$

where

$$\mathbf{F}_0 \quad = \quad -[\mathbf{I}_m \; \mathbf{0}_m \; \cdots \; \mathbf{0}_m](\mathbf{G}^T\mathbf{G} + \mathbf{\Lambda})^{-1}\mathbf{G}^T\mathbf{H}$$

Furthermore, assuming steady-state value $\mathbf{y}_\infty$ of the output becomes equal to the target value $\mathbf{r}$, $\mathbf{x}_\infty$ and $\mathbf{u}_\infty$ are given by the following equation.

$$\begin{bmatrix} \mathbf{x}_\infty \\ \mathbf{u}_\infty \end{bmatrix} = \begin{bmatrix} \mathbf{A}-\mathbf{I} & \mathbf{B} \\ \mathbf{C} & \mathbf{0} \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{0} \\ \mathbf{r} \end{bmatrix} \tag{11}$$

From Eq.(11), it can calculate the second and the third term of the right side of Eq.(10) as follows.

$$-\mathbf{F}_0\mathbf{x}_\infty + \mathbf{u}_\infty \quad = \quad -\left\{\mathbf{C}(\mathbf{A} - \mathbf{I} + \mathbf{BF}_0)^{-1}\mathbf{B}\right\}^{-1}\mathbf{r}$$

Then the control law is expressed by the following equation.

$$\mathbf{u}[k] \quad = \quad \mathbf{F}_0\mathbf{x}[k] + \mathbf{H}_0\mathbf{r} \tag{12}$$

where

$$\mathbf{H}_0 = -\left\{\mathbf{C}(\mathbf{A} - \mathbf{I} + \mathbf{BF}_0)^{-1}\mathbf{B}\right\}^{-1} \tag{13}$$

3. CONSTRUCTION OF TWO DOF GPC

The following control law is considered by including integral compensation in order to construct two DOF GPC.

$$\mathbf{u}[k] = \mathbf{F}_0\mathbf{x}[k] + \mathbf{H}_0\mathbf{r} + \mathbf{G}_0\mathbf{z}[k] \tag{14}$$

Where $\mathbf{z}[k]$ is integral compensation, $\mathbf{G}_0$ is integral gain. The control law (Eq.(14)) can make the steady state error zero. However without disturbance and modeling error, it may lead to increase in the control input and delay of tracking the target value in the transient response. So we make the effect of integral compensation appear only when the disturbance and modeling error exist. Also as $\mathbf{e}[k] = \mathbf{r} - \mathbf{y}[k]$, which is the difference between the target value and the output, is used for integral compensation as follows.

$$\mathbf{w}[k] \quad = \quad \frac{1}{\Delta}\mathbf{e}[k] \qquad (\Delta = 1 - z^{-1}) \tag{15}$$

In the case that the disturbance or modeling error does not exist, Eq.(14) should be epual to Eq.(13), in order to derive two DOF GPC law. Therefore the following equation is considered by substituting Eq.(13) into Eq.(1), and subtracting $\mathbf{x}[k]$ from both sides.

$$\mathbf{x}[k+1] - \mathbf{x}[k] \quad = \quad (\mathbf{A} - \mathbf{I} + \mathbf{BF}_0)\mathbf{x}[k] + \mathbf{BH}_0\mathbf{r}$$

Then

$$\begin{aligned} \mathbf{x}[k] \quad = \quad & (\mathbf{A} - \mathbf{I} + \mathbf{BF}_0)^{-1}\mathbf{x}[k+1] \\ & -(\mathbf{A} - \mathbf{I} + \mathbf{BF}_0)^{-1}\mathbf{x}[k] \\ & -(\mathbf{A} - \mathbf{I} + \mathbf{BF}_0)^{-1}\mathbf{BH}_0\mathbf{r} \end{aligned} \tag{16}$$

By substituting the above equation into $\mathbf{e}$[k] and using Eq.(12), we have.

$$\mathbf{e}[k] \quad = \quad -\mathbf{C}(\mathbf{A} - \mathbf{I} + \mathbf{BF}_0)^{-1}(\mathbf{x}[k+1] - \mathbf{x}[k])$$

That is, if the disturbance and modeling error do not exist, the integral compensation by the tracking error can be calculated as follows.

$$\begin{aligned} \mathbf{w}'[k] \quad &= \quad \frac{1}{\Delta}\mathbf{e}[k] \\ &= \quad -\mathbf{C}(\mathbf{A} - \mathbf{I} + \mathbf{BF}_0)^{-1}(\mathbf{A} + \mathbf{BF}_0) \\ & \qquad \cdot(\mathbf{x}[k] - \mathbf{x}[0]) \end{aligned} \tag{17}$$

Where $\mathbf{w}'[0]$ is assumed to be 0. Since Eq.(17) represents the integral compensation when the disturbance does not exist, the integral compensation $\mathbf{z}[k]$ of two DOF GPC is given by the following equation.

$$\mathbf{z}[k] \quad = \quad \mathbf{w}[k] - \mathbf{w}'[k]$$

From this equation, it always becomes $\mathbf{z}[k] = \mathbf{0}$ if the disturbance and modeling error does not exist, that is, the effect of the integral compensation does not appear. Thus, the control law of two DOF GPC is given.

4. SIMULATION

4.1. Derivation of Model

Firstly the following model shown in Fig.1 is considered.

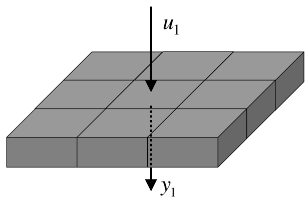

Fig. 1 Model of aluminum plate

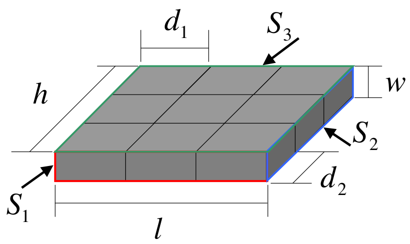

Fig. 2 Model of Parameter

The parameters of the aluminum plate thermal process model are given in Table.1

Density of aluminum: $\rho = 2700[\mathrm{kg/m^3}]$
Specific Heat: $c = 0.917[\mathrm{kJ/kgK}]$
Heat transfer coefficient: $k = 20[\mathrm{W/m^2K}]$
Thermal conductivity: $\lambda_f = 238[\mathrm{W/mK}]$
Height and Length: $l = 250[\mathrm{mm}]$
Width: $w = 10[\mathrm{mm}]$

Table 1 Parameters for the model

The state variables are defined as the following.

$$x_n = T_n - T_o \tag{18}$$

This model is nine dimensional model with dividing into 3×3 lengthwise and crosswise and $n = 1, 2, \cdots, 9$ to the right from the upper left. Where T_n is temperature of each part of the aluminum plate, T_o is ambient temperature. Then, three laws are used in the derivation of the model.
Fourier's law of heat conduction is given by,

$$q = -\lambda_f \frac{d\theta}{dn} \tag{19}$$

Where q is heat flow ratio $[\mathrm{W/m^2}]$, λ_f is thermal conductivity $[\mathrm{W/mK}]$, $d\theta/dn$ is temperature gradient $[\mathrm{K/m}]$
Newton's law of cooling is given by,

$$q = h(\theta_s - \theta_f) \tag{20}$$

h is heat transfer coefficient $[\mathrm{W/m^2K}]$.
The law of heat conduction is given by,

$$dQ = mc \cdot d\theta \tag{21}$$

c is specific heat $[\mathrm{J/kgK}]$, m is mass of each part $[\mathrm{kg}]$.
Then the following equations of the system are obtained from the state variables of Eq.(18) and the laws of Eq.(19),(20) and (21).

$$\begin{aligned}
mc\frac{dx_1}{dt} &= -(k(\frac{1}{3}S_1 + \frac{1}{3}S_2 + \frac{2}{9}S_3) \\
&\quad +\lambda_f(\frac{S_1}{d_1} + \frac{S_2}{d_2}))x_1 \\
&\quad +(\lambda_f\frac{S_2}{d_2})x_2 + (\lambda_f\frac{S_1}{d_1})x_4 \\
mc\frac{dx_2}{dt} &= -(k(\frac{1}{3}S_1 + \frac{2}{9}S_3) \\
&\quad +\lambda_f(\frac{S_1}{d_1} + 2\frac{S_2}{d_2}))x_2 + (\lambda_f\frac{S_2}{d_2})x_1 \\
&\quad +(\lambda_f\frac{S_2}{d_2})x_3 + (\lambda_f\frac{S_1}{d_1})x_5 \\
mc\frac{dx_3}{dt} &= -(k(\frac{1}{3}S_1 + \frac{1}{3}S_2 + \frac{2}{9}S_3) \\
&\quad +\lambda_f(\frac{S_1}{d_1} + \frac{S_2}{d_2}))x_3 \\
&\quad +(\lambda_f\frac{S_2}{d_2})x_2 + (\lambda_f\frac{S_1}{d_1})x_6 \\
mc\frac{dx_4}{dt} &= -(k(\frac{1}{3}S_2 + \frac{2}{9}S_3) \\
&\quad +\lambda_f(2\frac{S_1}{d_1} + \frac{S_2}{d_2}))x_4 + (\lambda_f\frac{S_1}{d_1})x_1 \\
&\quad +(\lambda_f\frac{S_2}{d_2})x_5 + (\lambda_f\frac{S_1}{d_1})x_7 \\
mc\frac{dx_5}{dt} &= -(k(\frac{2}{9}S_3) + \lambda_f(2\frac{S_1}{d_1} + 2\frac{S_2}{d_2}))x_5 \\
&\quad +u_1 + (\lambda_f\frac{S_1}{d_1})x_2 + (\lambda_f\frac{S_2}{d_2})x_4 \\
&\quad +(\lambda_f\frac{S_2}{d_2})x_6 + (\lambda_f\frac{S_1}{d_1})x_8 \\
mc\frac{dx_6}{dt} &= -(k(\frac{1}{3}S_2 + \frac{2}{9}S_3) + \\
&\quad \lambda_f(2\frac{S_1}{d_1} + \frac{S_2}{d_2}))x_6 + (\lambda_f\frac{S_1}{d_1})x_3 \\
&\quad +(\lambda_f\frac{S_2}{d_2})x_5 + (\lambda_f\frac{S_1}{d_1})x_4 \\
mc\frac{dx_7}{dt} &= -(k(\frac{1}{3}S_1 + \frac{1}{3}S_2 + \frac{2}{9}S_3) \\
&\quad +\lambda_f(\frac{S_1}{d_1} + \frac{S_2}{d_2}))x_7 \\
&\quad +(\lambda_f\frac{S_1}{d_1})x_4 + (\lambda_f\frac{S_2}{d_2})x_8 \\
mc\frac{dx_8}{dt} &= -(k(\frac{1}{3}S_1 + \frac{2}{9}S_3) \\
&\quad +\lambda_f(\frac{S_1}{d_1} + 2\frac{S_2}{d_2}))x_8 + (\lambda_f\frac{S_1}{d_1})x_5 \\
&\quad +(\lambda_f\frac{S_2}{d_2})x_7 + (\lambda_f\frac{S_2}{d_2})x_9
\end{aligned}$$

$$
\begin{aligned}
mc\frac{dx_9}{dt} &= -(k(\frac{1}{3}S_1 + \frac{1}{3}S_2 + \frac{2}{9}S_3) \\
&\quad + \lambda_f(\frac{S_1}{d_1} + \frac{S_2}{d_2}))x_9 \\
&\quad + (\lambda_f\frac{S_1}{d_1})x_6 + (\lambda_f\frac{S_2}{d_2})x_8
\end{aligned}
$$

From these equations, the state space equations were obtained. For the sake of simple expression of the coefficients for the above equations, each coefficient of the equation for $x_i(i = 1, \cdots, 9)$ is renamed as $a_{ij}(i = 1, \cdots, 9, j = 1, \cdots, 9)$.

$$
\begin{aligned}
mc\frac{dx_1}{dt} &= a_{11}x_1 + a_{12}x_2 + a_{14}x_4 \\
mc\frac{dx_2}{dt} &= a_{22}x_2 + a_{21}x_1 + a_{23}x_3 + a_{25}x_5 \\
mc\frac{dx_3}{dt} &= a_{33}x_3 + a_{32}x_2 + a_{36}x_6 \\
mc\frac{dx_4}{dt} &= a_{44}x_4 + a_{41}x_1 + a_{45}x_5 + a_{47}x_7 \\
mc\frac{dx_5}{dt} &= a_{55}x_5 + a_{52}x_2 + a_{54}x_4 + a_{56}x_6 + a_{58}x_8 + u_1 \\
mc\frac{dx_6}{dt} &= a_{66}x_6 + a_{63}x_3 + a_{65}x_5 + a_{69}x_9 \\
mc\frac{dx_7}{dt} &= a_{77}x_7 + a_{74}x_4 + a_{78}x_8 \\
mc\frac{dx_8}{dt} &= a_{88}x_8 + a_{85}x_5 + a_{87}x_7 + a_{89}x_9 \\
mc\frac{dx_9}{dt} &= a_{99}x_9 + a_{96}x_6 + a_{98}x_8
\end{aligned}
$$

The result of the above state space equations is given.

$$
\begin{aligned}
\mathbf{x}(k+1) &= \mathbf{A}\mathbf{x}(k) + \mathbf{B}\mathbf{u}(k) \\
\mathbf{y}(k) &= \mathbf{C}\mathbf{x}(k)
\end{aligned}
$$

The system matrices **A**, **B** and **C** are defined as follows,

$$
\mathbf{A} = \frac{1}{mc}\begin{bmatrix}
a_{11} & a_{12} & 0 & a_{14} & & & & & \\
a_{21} & a_{22} & a_{23} & 0 & a_{25} & & & \mathbf{0} & \\
0 & a_{32} & a_{33} & 0 & 0 & a_{36} & & & \\
a_{41} & 0 & 0 & a_{44} & a_{45} & 0 & a_{47} & & \\
 & a_{52} & 0 & a_{54} & a_{55} & a_{56} & 0 & a_{58} & \\
 & & a_{63} & 0 & a_{65} & a_{66} & 0 & 0 & a_{69} \\
 & \mathbf{0} & & a_{74} & 0 & 0 & a_{77} & a_{78} & 0 \\
 & & & & a_{85} & 0 & a_{87} & a_{88} & a_{89} \\
 & & & & & a_{96} & 0 & a_{98} & a_{99}
\end{bmatrix}
$$

$$
\mathbf{B} = \frac{1}{mc}\begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}^T
$$

$$
\mathbf{C} = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}
$$

$$
\mathbf{x} = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 & x_8 & x_9 \end{bmatrix}^T
$$

Where the sampling time is 20[s]. Then the state space representation and the derivation of the aluminum plate temperature control model in the nine dimensional has been completed. It is noticed that although we assume that the obtained model is single input single output system and the positions of input and output are $n = 5$ (located in the center part of the model), their positions can be changed easily.

4.2. Confirmation of the Model

In order to check the validity of the model, the step response for the model which is determined in section 4, is checked. In this case, it was confirmed spread of heat is weather it is natural. From Fig.7 to Fig.12, the temperature increase and their distributions are shown in the case that the heat input is applied at each part. At this time, the amount of heat input is always constant and are applying with 15[W]. Further, the output when the inputs to n_1, n_3, n_7 and n_9 is confirmed by Fig.7. When the inputs to n_2, n_4, n_6 and n_8 were given, the result is shown in Fig.9. Moreover, when the input to n_5 was given, the result is shown in Fig.11. Also, Fig.8, 10 and 12 show the temperature distribution of each part (in the case for the input to n_1, n_2 or n_5) at $t = 500$[s], 1000[s] and 4500[s].

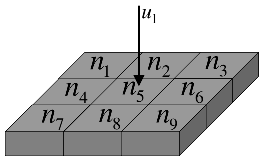

Fig. 3 The Simulation Model of Aluminum

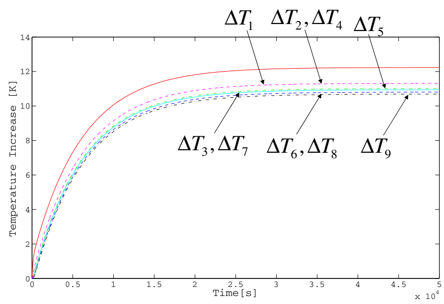

Fig. 4 Temperature increase(input to n_1, n_3, n_7 and n_9)

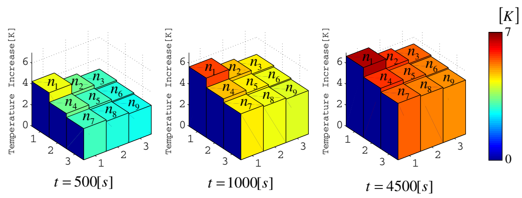

Fig. 5 Temperature distribution (input to n_1)

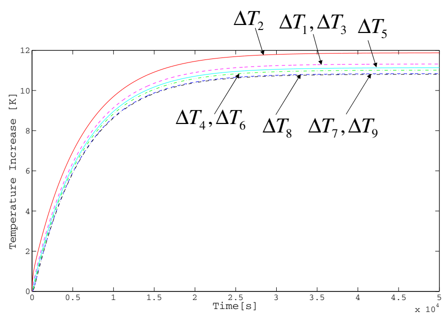

Fig. 6 Temperature increase(input to n_2, n_4, n_6 and n_8)

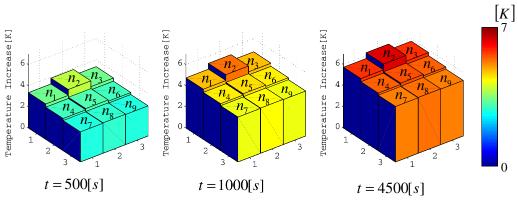

Fig. 7 Temperature Distribution(input to n_2)

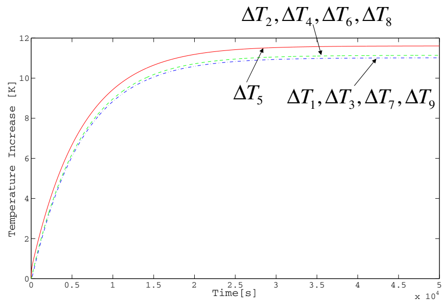

Fig. 8 Temperature increase(input to n_5)

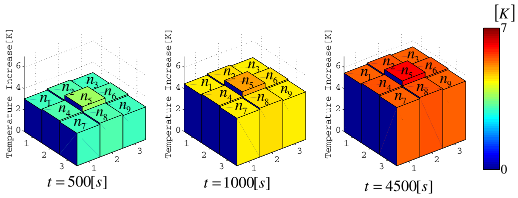

Fig. 9 Temperature distribution(input to n_5)

It found that the heat spreads concentrically from the input point based on the results in Fig.7, 9 and 11 and the results in Fig.8, 10 and 12. Therefore, it can find that this model is appropriate.

4.3. Expansion in the Height Direction

In order to derive three dimensional model of aluminum plate thermal process system, based on the model of previous section, the extension to the vertical direction for the aluminum plate thermal process is considered in this section. Fig.13 shows the extended model and Fig.14, 15 and 16 give the results of temperature increase.

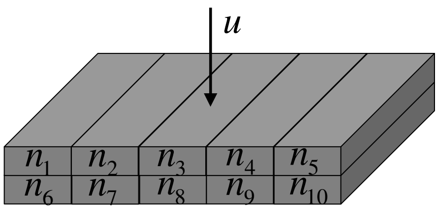

Fig. 10 The Simulation Model of Aluminum

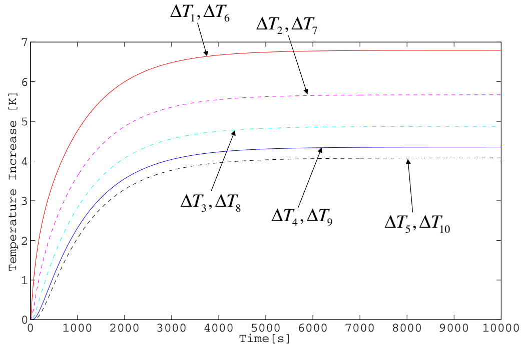

Fig. 11 Temperature increase(input to n_1)

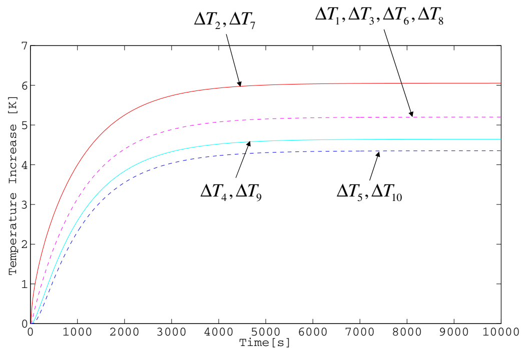

Fig. 12 Temperature increase(input to n_2)

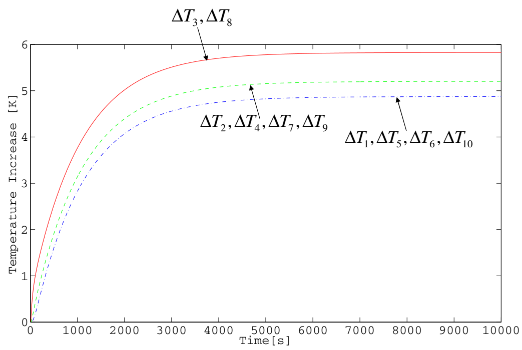

Fig. 13 Temperature increase(input to n_3)

Although there is no temperature difference between n_i and $n_{i+5}(i = 1, \cdots, 5)$, it seems to be caused by thinness of the aluminum plate model. But from Fig.14, 15 and 16, the temperature increase has a tendency that closer position is to the input position, the higher temperature appears. Therefore it can find that the extended model is also appropriate.

5. APPLICATION OF THE CONTROL LAW

On the model obtained in the previous section, the predictive control systems for the three cases are applied. Control target of temperature is to increase $4[\mathrm{K}]$ from ambient temperature. In addition, the disturbanse temperature $0.5[\mathrm{K}]$ is added at $t = 5000[\mathrm{s}]$. The design parameters of GPC are given in Table.2

$N_1 = 10$
$N_2 = 120$
$N_u = 2$
$\mathbf{\Lambda} = 0.01\mathbf{I}$
$\mathbf{G}_0 = 0.2\mathbf{I}$

Table 2 Parameters for GPC

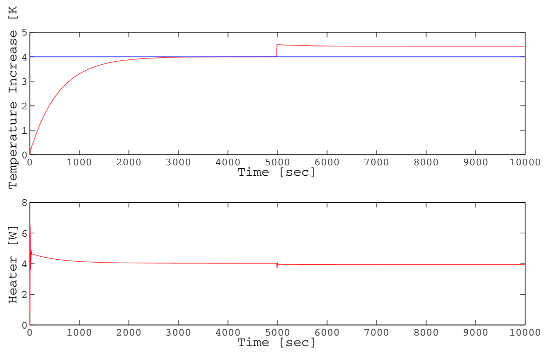

Fig. 14 Simulation result by GPC with no integration (Eq.(13))

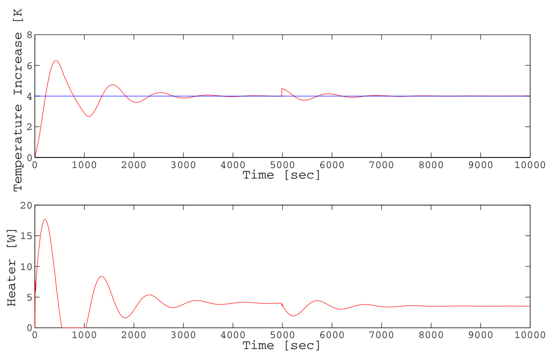

Fig. 15 Simulation result by GPC with integration ($\mathbf{z}[k] = \mathbf{w}[k]$)

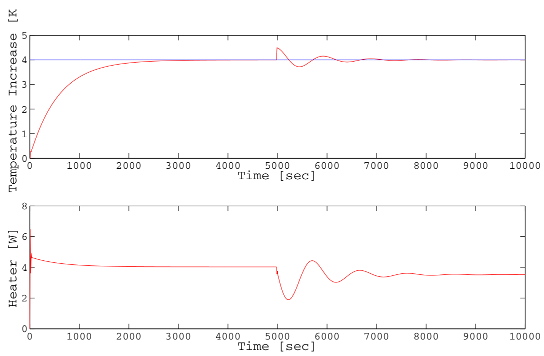

Fig. 16 Simulation result by two DOF GPC

From these results, it can find that two DOF GPC can achieve the target tracking and reduce the overshoot when disturbance is added as confirmed in the previous studies [6].

6. CONCLUSION

This paper extended the aluminum plate temperature control experimental device model in two dimensional model from one dimensional model which is used in the previous study. The simulation was conducted in order to verify the validity of the model. That is, the figures for temperature increase and the color maps for temperature distribution are given to shown the relations between input and output. As future works, the model in three dimensions must be constructed and its simulation is investigated in order to confirm the validity of the model. Moreover the number of divisions of the model should increase because of aiming at accuracy of calculating result.

REFERENCES

[1] D. W. Clarke, C. Mohtadi and P. S. Tuffs, "Generalized Predictive Control-Part I.The Basic Algorithm," *Automatica*, Vol. 23, No. 2, pp. 137-148, 1987.

[2] E. F. Cawacho and C. Bordons, "Model Predictive Control in the Process Industry," Springer, 1995.

[3] Satoshi Okazaki, Junki Nishizaki, Akira Yanou, Mamoru Minami and Mingcong Deng, "Strongly Stable Generalized Predictive Control Focused on Closed-Loop Characteristics," *Transactions of SICE*, Vol.47 No. 7, pp.317-325 (2011)

[4] Junki Nishizaki, "A Design Method of Strongly Stable Generalized Predictive Control System using Coprime Factorization," *Graduate thesis of Department of System and Engineering*, Okayama University, 2011.

[5] Akira Yanou, Shiro Masuda, Mingcong Deng and Akira Inoue, "An Extension of Self-Tuning Two Degree-of-Freedom GPC Based on Polynomial Approach with Computational Savings," *International Journal of Innovative Computing, Information and Control*, Vol.5, No.10(B), pp. 3431-3438, 2009.

[6] Naoki Hosoya, Akira Yanou, Mamoru Minami and Takayuki Matsuno, "Application of Two Degree-of-Freedom Generalized Predictive Control to Thermal Model for Two Dimensional Aluminum Plate," Proceedings of the SCI'14 (in Japanese) (2014, to be presemted)

[7] Zhang Wenlong, Masao Imaeda, Kyouji Hashimoto, Takaaki Kanbe and Hirofumi Hattori, "Temperature Control in Distillation Column Using Generalized Predictive Control Method," *Kagaku Kogaku Ronbunshu*, 29, pp.541-545 (2003)

[8] Ikuo Nanno, Masahito Tanaka, Nobutomo Matsunaga and Shigeyasu Kawaji "Development of the Gradient Temperature Control Method for Temperature Equalization -A new approach to decoupling control based on PID control-," *IEEJ Transactions on Electronics, Information and Systems*, Vol. 124, No. 8, pp.1606-1612 (2004)