ADCONIP 2014 Hiroshima

Design Method of Generalized Minimum Variance Control Using Strong Stability Rate

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***Abstract*— This paper proposes a design method of generalized minimum variance control (GMVC) using strong stability rate. Strong stability rate is a novel and simple concept for plant safety, and it is defined by ratio of the open-loop gain and the closed-loop gain of controlled system through Youla-Kucera parametrization. For safety, it is desirable that the open-loop output is closer to the closed-loop output even if feedback signal becomes zero by accident. In other words, ratio of open-loop gain and closed-loop gain plays an important role for safety. Therefore this paper extends GMVC using coprime factorization and designs strongly stable GMVC with strong stability rate. A numerical example is given in order to verify the validity of the proposed method.**

I. Introduction

Generalized Minimum Variance Control (GMVC) has been proposed by Clarke and others[1]. GMVC is one of the control methods for application in industry. For example, GMVC can be used for a design method of PID controller's gains[2], [3]. Moreover the closed-loop stability of self-tuning GMVC with white noise has been considered and assured[4]. In order to design GMVC systems, generalized output is selected for the closed-loop system to be stable. Then the controller is designed to minimize the variance of the generalized output, and it cannot be designed independently of the closed-loop system. In case of application for industry, it is desirable that both closed-loop system and controller are stable for safety. In other words, even if closed-loop characteristic is designed, it is desirable that the characteristic of controller can be designed to be stable independently.

Authors have proposed extended GMVC design method[5], [6]. The extended method introduces a new design parameter for conventional GMVC by using Youla-Kucera parameterization[7]. In the method, the poles of controller can be designed by the newly introduced parameter and can be chosen independently of the poles of closed-loop system. Therefore if controller is designed to be stable, a strongly stable system can be obtained, which means that both closed-loop system and controller are stable. Although the authors have proposed design methods[8], [9], [10], [11], [12] by using coprime factorization approach and showed that strongly stable system can be obtained, the open-loop output was not considered clearly. Under the assumption that the controlled plant is stable, the stable open-loop output has a possibility that its value strays out of reference signal largely when feedback signal becomes zero by accident, even if the obtained system is strongly stable. This situation causes abnormal rise in temperature for thermal process or overflow for tank system. That is, safety for strongly stable system has not been considered adequately.

For this problem, concept of strong stability rate has been proposed[13], [15], [16] and a design method of generalized predictive control system using it has also been explored[14]. Strong stability rate is defined by ratio of the open-loop gain and the closed-loop gain of controlled system through Youla-Kucera parametrization. And it can assess the safety of controlled system in the meaning of whether the open-loop gain is close to the closed-loop gain or not. If best value of strong stability rate is given, open-loop output of controlled system does not stray out of reference signal. In this case, for example, abnormal rise in temperature for thermal process or overflow for tank system does not occur because the open-loop output keeps at reference signal even if feedback signal becomes zero by accident. Therefore this paper proposes a design method of GMVC system using strong stability rate through coprime factorization. In this paper strongly stable system based on strong stability rate is obtained. A numerical example is given in order to verify the validity of the proposed method.

This paper is organized as follows. Section 2 describes problem statement and conventional GMVC. Section 3 shows concept of strong stability rate using coprime factorization. In section 4 GMVC system design through strong stability rate is proposed. In section 5 a numerical example is presented. Section 6 summarizes the result of this paper.

<u>Notations</u> This paper assumes that the controlled plant is stable. z^{-1} means backward shift operator $z^{-1}y(t) = y(t-1)$. $A[z^{-1}]$ and $A(z^{-1})$ means polynomial and rational function with z^{-1} respectively. Steady state gain $A(1)$ of transfer function is calculated as $z^{-1} = 1$ under the assumption that signals such as input and output for system does not change with regard to time t.

II. Problem Statement

The following single-input single-output system is considered.

$$A[z^{-1}]y(t) = z^{-k_m}B[z^{-1}]u(t) + C[z^{-1}]\xi(t) \qquad (1)$$
$$t = 0, 1, 2\cdots$$

Where $u(t)$ and $y(t)$ are input and output respectively. k_m is time delay, $\xi(t)$ is a white Gaussian noise with zero mean.

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$A[z^{-1}]$, $B[z^{-1}]$ and $C[z^{-1}]$ are the polynomials with degrees n, m and l.

$$\begin{aligned} A[z^{-1}] &= 1 + a_1 z^{-1} + \cdots a_n z^{-n} \\ B[z^{-1}] &= b_0 + b_1 z^{-1} + \cdots b_m z^{-m} \\ C[z^{-1}] &= 1 + c_1 z^{-1} + \cdots c_l z^{-l} \end{aligned} \quad (2)$$

On the system (1) the following assumptions are hold.

[A.1] The degrees n, m and l of $A[z^{-1}]$, $B[z^{-1}]$ and $C[z^{-1}]$, and the time delay k_m are known.

[A.2] The coefficients of $A[z^{-1}]$, $B[z^{-1}]$ and $C[z^{-1}]$ are known.

[A.3] The polynomials $A[z^{-1}]$ and $B[z^{-1}]$, $A[z^{-1}]$ and $C[z^{-1}]$ are coprime.

[A.4] The polynomial $C[z^{-1}]$ is stable.

The control objective is to make the output $y(t)$ track the reference signal $w(t)$. To achieve this objective, the following performance index J averaged over the noise is minimized;

$$\begin{aligned} \Phi(t+k_m) &= P[z^{-1}]y(t+k_m) + Q[z^{-1}]u(t) \\ &\quad -R[z^{-1}]w(t) \quad (3) \\ J &= \mathrm{E_X}[\Phi^2(t+k_m)] \quad (4) \end{aligned}$$

where $\Phi(t+k_m)$ is called as generalized output. $P[z^{-1}]$, $Q[z^{-1}]$ and $R[z^{-1}]$ are polynomials with degrees of n_p, n_q and n_r. These polynomials are selected to obtain stable desirable closed-loop poles. In conventional GMVC, Diophantine equation is given for solutions $E[z^{-1}]$ and $F[z^{-1}]$.

$$P[z^{-1}]C[z^{-1}] = A[z^{-1}]E[z^{-1}] + z^{-k_m}F[z^{-1}] \quad (5)$$

where

$$\begin{aligned} E[z^{-1}] &= 1 + e_1 z^{-1} + \cdots + e_{k_m-1} z^{-(k_m-1)} \quad (6) \\ F[z^{-1}] &= f_0 + f_1 z^{-1} + \cdots + f_{n_1} z^{-n_1} \quad (7) \\ n_1 &= \max\{n-1, n_p + l - k_m\} \quad (8) \end{aligned}$$

The solution $E[z^{-1}]$ is used to calculate the following polynomial $G[z^{-1}]$. $T[z^{-1}]$ gives the closed-loop characteristics.

$$\begin{aligned} G[z^{-1}] &= E[z^{-1}]B[z^{-1}] + C[z^{-1}]Q[z^{-1}] \quad (9) \\ T[z^{-1}] &= P[z^{-1}]B[z^{-1}] + Q[z^{-1}]A[z^{-1}] \quad (10) \end{aligned}$$

From (5) and (9), the generalized output and its prediction are given.

$$\begin{aligned} \Phi(t+k_m) &= \hat{\Phi}(t+k_m|t) + E[z^{-1}]\xi(t+k_m) \quad (11) \\ \hat{\Phi}(t+k_m|t) &= (F[z^{-1}]y(t) + G[z^{-1}]u(t) \\ &\quad -C[z^{-1}]R[z^{-1}]w(t))/C[z^{-1}] \quad (12) \end{aligned}$$

Since the prediction $\hat{\Phi}(t+k_m|t)$ and the noise term $E[z^{-1}]\xi(t+k_m)$ have no correlation each other, the control law $u(t)$ to minimize J can be obtained by the following equation.

$$\hat{\Phi}(t+k_m|t) = 0 \quad (13)$$

Then the control law is obtained as,

$$u(t) = \frac{C[z^{-1}]R[z^{-1}]}{G[z^{-1}]}w(t) - \frac{F[z^{-1}]}{G[z^{-1}]}y(t) \quad (14)$$

The closed-loop system can be given by (14).

$$y(t) = \frac{z^{-k_m}B[z^{-1}]R[z^{-1}]}{T[z^{-1}]}w(t) + \frac{G[z^{-1}]}{T[z^{-1}]}\xi(t) \quad (15)$$

Where $T[z^{-1}]$ is defined in (10)

III. Strong Stability Rate

This section shows the concept of strong stability rate and the preliminary.

A. Coprime Factorization of Controlled Systems

For coprime factorization approach, the family of stable rational functions RH_∞ is considered;

$$\begin{aligned} RH_\infty = \{G(z^{-1}) = \frac{G_n[z^{-1}]}{G_d[z^{-1}]}, \\ G_d[z^{-1}]\text{: stable polynomial}\} \end{aligned}$$

Transfer function $G_p(z^{-1})$ is given in the form of a ratio of rational functions in RH_∞,

$$\begin{aligned} y(t) &= G_p(z^{-1})u(t) \\ &= N(z^{-1})D^{-1}(z^{-1})u(t) \quad (16) \end{aligned}$$

$N(z^{-1})$ and $D(z^{-1})$ are rational functions in RH_∞ and coprime each other. This paper assumes that the controlled system $G_p(z^{-1})$ is stable.

Next, the following Bezout identity is considered.

$$X(z^{-1})N(z^{-1}) + Y(z^{-1})D(z^{-1}) = 1 \quad (17)$$

Where the solutions $X(z^{-1})$ and $Y(z^{-1})$ of Bezout identity are in RH_∞. Then all the stabilizing compensator is given in Youla-Kucera parameterization[7] from (16) and (17).

$$\begin{aligned} u(t) &= C_1(z^{-1})w(t) - C_2(z^{-1})y(t) \quad (18) \\ C_1(z^{-1}) &= (Y(z^{-1}) - U(z^{-1})N(z^{-1}))^{-1}K(z^{-1}) \quad (19) \\ C_2(z^{-1}) &= (Y(z^{-1}) - U(z^{-1})N(z^{-1}))^{-1} \\ &\quad \cdot (X(z^{-1}) + U(z^{-1})D(z^{-1})) \quad (20) \end{aligned}$$

$U(z^{-1}),\ K(z^{-1}) \in RH_\infty$ are design parameters and $w(t)$ is reference signal. From (18), (19), (20) and (16), the closed-loop transfer function is given.

$$\begin{aligned} y(t) &= N(z^{-1})D^{-1}(z^{-1})(Y(z^{-1}) - U(z^{-1}) \\ &\quad \cdot N(z^{-1}))^{-1}K(z^{-1})w(t) - N(z^{-1})D^{-1}(z^{-1}) \\ &\quad \cdot (Y(z^{-1}) - U(z^{-1})N(z^{-1}))^{-1}(X(z^{-1}) \\ &\quad + U(z^{-1})D(z^{-1}))y(t) \quad (21) \end{aligned}$$

Therefore

$$\begin{aligned} &\{D(z^{-1})(Y(z^{-1}) - U(z^{-1})N(z^{-1})) + N(z^{-1}) \\ &\cdot (X(z^{-1}) + U(z^{-1})D(z^{-1}))\}y(t) \\ &\quad = N(z^{-1})K(z^{-1})w(t) \quad (22) \end{aligned}$$

Then the closed-loop transfer function is given from (17).

$$y(t) = N(z^{-1})K(z^{-1})w(t) \quad (23)$$

If the compensator is designed for settling control, the output $y(t)$ converges to $w(t)$ as time progresses. It means that the steady-state gain of closed-loop system (23) is designed to be $N(1)K(1)=1$. Moreover the design parameter $U(z^{-1})$ in the stabilizing compensator (18) does not affect (23). Therefore when closed-loop system (23) is designed to be stable and stabilizing compensator (18) is also designed to be stable through $U(z^{-1})$, Strongly stable system can be obtained. Here it is noticed that the closed-loop system (23) is independent of design parameter $U(z^{-1})$.

B. Concept of Strong Stability Rate

In the previous research[17], although the authors have proposed a design method of strongly stable system and given the selection method of design parameter U, which can equate steady state gains of the closed-loop system and the open-loop system, the condition of U is rigid for designing strongly stable system and there was a case that strongly stable system could not be obtained. Therefore the authors relaxed the condition of U in place of not equating the steady state gains of the closed-loop and open-loop systems. Moreover, for one of the safety indices, strong stability rate[13], [15] has been defined as nearness of the steady state gains in the assumption that a strongly stable system was obtained.

In this subsection the concept is described briefly. When considering the open-loop system for the closed-loop system (23) (assuming that the feedback signal $C_2(z^{-1})y(t)$ in the stabilizing compensator (18) becomes zero), the compensator (18) is given as follows.

$$u(t) = (Y(z^{-1})-U(z^{-1})N(z^{-1}))^{-1}K(z^{-1})w(t) \quad (24)$$

From (16), the open-loop transfer function from w to y is given as,

$$\begin{aligned} y(t) &= N(z^{-1})D^{-1}(z^{-1})u(t) \\ &= N(z^{-1})D^{-1}(z^{-1})(Y(z^{-1})-U(z^{-1}) \\ &\quad \cdot N(z^{-1}))^{-1}K(z^{-1})w(t) \\ &= (Y(z^{-1})D(z^{-1})-U(z^{-1})N(z^{-1})D(z^{-1}))^{-1} \\ &\quad \cdot N(z^{-1})K(z^{-1})w(t) \end{aligned} \quad (25)$$

Because of $Y(z^{-1})D(z^{-1})=1-X(z^{-1})N(z^{-1})$, the open-loop system can be obtained as the following equation.

$$\begin{aligned} y(t) &= (1-X(z^{-1})N(z^{-1})-U(z^{-1})N(z^{-1}) \\ &\quad \cdot D(z^{-1}))^{-1}N(z^{-1})K(z^{-1})w(t) \\ &= \{1-(X(z^{-1})+U(z^{-1})D(z^{-1}))N(z^{-1})\}^{-1} \\ &\quad \cdot N(z^{-1})K(z^{-1})w(t) \end{aligned} \quad (26)$$

Therefore the steady state of this system is given.

$$\begin{aligned} y(t) &= \{1-(X(1)+U(1)D(1))N(1)\}^{-1} \\ &\quad \cdot N(1)K(1)w(t) \end{aligned} \quad (27)$$

From (23) and (27), defining strong stability rate s as ratio of steady state gains of closed-loop and open-loop systems, the following definition is given.

$$\begin{aligned} s &= \frac{\{1-(X(1)+U(1)D(1))N(1)\}^{-1}N(1)K(1)}{N(1)K(1)} \\ &= \{1-(X(1)+U(1)D(1))N(1)\}^{-1} \end{aligned} \quad (28)$$

It can find that strong stability rate s is equal to the steady state gain of the open-loop system, and the design parameter $U(1)$ can design the poles of compensator (18) without changing the closed-loop poles of (23). In the case of $s=1$ the steady state gain of open-loop system becomes equal to the closed-loop's one, even if the feedback signal $C_2(z^{-1})y(t)$ in (18) becomes zero. That is, the open-loop system's output becomes equal to the reference signal $w(t)$ in steady state. For this case, it means that the strongly stable system gives the highest safety. On the other hands, in the case of $s\neq 1$, the open-loop system's output converges to a constant value but deviates from $w(t)$. For example, this case implies possibility for abnormal temperature increase of temperature control systems or overflow of tank systems. Therefore strong stability rate s means that $s=1$ is best for safety and $s\neq 1$ shows a decline in safety.

In the next section generalized minimum variance control system is designed through strong stability rate under the assumption that the controlled plant is stable and the strongly stable system is obtained by design parameter $U(1)$.

IV. GMVC System Design Through Strong Stability Rate

In case that $P[z^{-1}]$ and $Q[z^{-1}]$ in generalized output are chosen for $T[z^{-1}]$ to be stable, comparing transfer function (16) to (15), $N(z^{-1})$ and $D(z^{-1})$ can be chosen as follows;

$$N(z^{-1}) = z^{-k_m}B[z^{-1}]/T[z^{-1}] \quad (29)$$

$$D(z^{-1}) = A[z^{-1}]/T[z^{-1}] \quad (30)$$

Substituting (29) and (30) into Bezout equation (17) and comparing it to Diophantine equation (5), the solutions $X(z^{-1})$ and $Y(z^{-1})$ of Bezout equation are given.

$$X(z^{-1}) = F[z^{-1}]/C[z^{-1}] \quad (31)$$

$$Y(z^{-1}) = G[z^{-1}]/C[z^{-1}] \quad (32)$$

Then the control law (14) is obtained from Youla-Kucera parameterization (18) by selecting the design parameters as,

$$K(z^{-1}) = R[z^{-1}] \quad (33)$$

$$U(z^{-1}) = 0 \quad (34)$$

To extend the controller (14), instead of choosing $U(z^{-1})$ as 0, it is used as a new design parameter $U(z^{-1})=U(1)$ for the controller (18). In the previous research[17] although the design parameter is given as $U(1)=-D^{-1}(1)X(1)$, whether strongly stable system can be obtained or not depends on the values of $D(1)$ or $X(1)$. As mentioned above, the proposed method in this paper uses the following parameter with arbitrary constant α in order to relax the condition of $U(1)$.

$$U(1) = -\alpha D^{-1}(1)X(1) \quad (35)$$

Then the extended controller through $U(1)$ is obtained as follows.

$$\begin{aligned} &(G[z^{-1}]T[z^{-1}]-U(1)z^{-k_m}B[z^{-1}]C[z^{-1}])u(t) \\ &\quad = C[z^{-1}]T[z^{-1}]R[z^{-1}]w(t)-(F[z^{-1}]T[z^{-1}] \\ &\qquad +U(1)A[z^{-1}]C[z^{-1}])y(t) \end{aligned} \quad (36)$$

If $\alpha = 1$ is selected, the design parameter becomes equal to [17]. Moreover in the case that the strongly stable system was obtained, strong stability rate becomes $s = 1$ because of (28). On the other side, if a strongly stable system was not obtained by $\alpha = 1$, α must be selected so that the compensator (18) becomes stable. In this case, strong stability rate s does not become 1. It means that the steady state output for the open-loop system deviates from the reference signal $w(t)$.

To calculate this control law, the polynomial operating on $u(t)$ in the left-hand side of (36) is divided by the leading term g_0 and the remaining term.

$$\begin{aligned} & G[z^{-1}]T[z^{-1}] - U(1)z^{-k_m}B[z^{-1}]C[z^{-1}] \\ & \qquad = \quad g_0 + z^{-1}G'[z^{-1}] \end{aligned} \tag{37}$$

Therefore the control law (36) is calculated by

$$\begin{aligned} u(t) \quad = \quad & \frac{1}{g_0}\{C[z^{-1}]T[z^{-1}]R[z^{-1}]w(t) \\ & -(F[z^{-1}]T[z^{-1}] + U(1)A[z^{-1}]C[z^{-1}])y(t) \\ & -G'[z^{-1}]u(t-1)\} \end{aligned} \tag{38}$$

From (23) it is noticed that the transfer function from reference signal to output is independent of $U(z^{-1})$. And the poles of compensator can be given by the following equation.

$$G[z^{-1}]T[z^{-1}] - U(1)z^{-k_m}B[z^{-1}]C[z^{-1}] = 0 \tag{39}$$

Therefore the extended controller can be re-designed not only to be stable but also to make strong stability rate s closer to 1 by selecting α in (35).

V. Numerical Example

In this section, the numerical example is shown to verify the validity of the proposed method. The following controlled system is considered.

$$\begin{aligned} A[z^{-1}] &= 1 + 0.6z^{-1} + 0.7z^{-2} \\ B[z^{-1}] &= 0.5 - 1.5z^{-1} \\ C[z^{-1}] &= 1, \quad k_m \; = \; 1 \end{aligned}$$

Simulation steps are 200, the initial values of output and input are assumed to be zero. The disturbance as the white Gaussian noise with the variance $\sigma^2 = 0.04$ is heuristically added to the output. In order to design the closed-loop characteristic to be stable, the generalized output is given so as to make the controlled output $y(t)$ track the reference signal $w(t) = 1$.

$$\Phi(t+1) \quad = \quad y(t+1) + 0.8u(t) - 0.84z^{-2} \cdot w(t)$$

The feedback signal $C_2(z^{-1})y(t)$ becomes zero after 140th steps. The absolute value of closed-loop poles is 0.6563. So the closed-loop system is designed to be stable. $U(1) = -\alpha D^{-1}(1)X(1)$ is calculated as 0.4273 for $\alpha = 0.9$. Then the absolute values of compensator's poles are 0.9217 and 0.585. That is, the compensator makes the strongly stable system. In this case the strong stability rate is $s = 1.1831$ and it means that the open-loop output in steady state becomes 1.1831 for the reference signal $w(t) = 1$. Therefore it finds that the new design parameter $U(1) = -\alpha D^{-1}(1)X(1)$ can construct a strongly stable system by α and the derived strongly stable system can be evaluated through strong stability rate s.

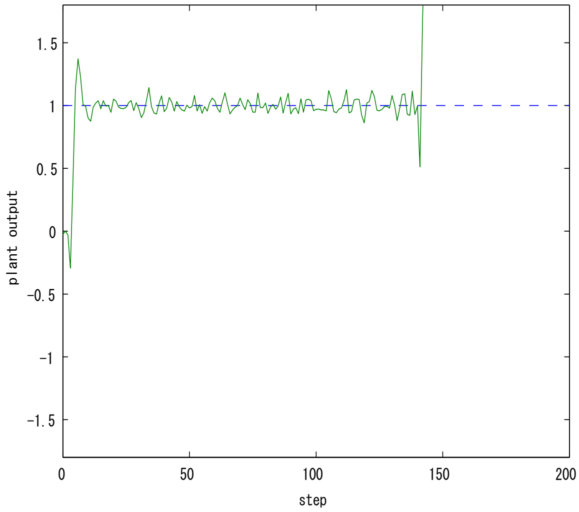

Fig. 1. Conventional GMVC (output)

When the new design parameter is selected as $U(1) = 0$, the compensator becomes the conventional GMVC (14). Then the absolute value of compensator's pole is 1.1538 although the closed-loop poles become equal to the proposed ones. It means that the conventional GMVC of this case does not make strongly stable system. From this point it finds that the new design parameter $U(1) = -\alpha D^{-1}(1)X(1)$ is effective to construct a strongly stable system evaluated by strong stability rate.

Fig.1 and Fig.3 show the plant outputs by the conventional GMVC and the proposed method. The broken lines of these figures show the reference signal. The solid lines of them show the plant output. Fig.2 and Fig.4 show the control input. When the feedback signal becomes zero after 140th step, the output and the input by the conventional GMVC diverge. On the other hand, the proposed method do not diverge because $U(1)$ described above gives the strongly stable system. Moreover, the different value of variance for disturbance ($\sigma^2 = 0.2$) was confirmed. Fig.5 and Fig.7 show the plant outputs by the conventional GMVC and the proposed method. In each figure, the broken line and the solid line mean reference signal and plant output respectively. Fig.6 and Fig.8 show the control inputs. Even if the feedback signal became zero after 140th step, their results did not change. That is, the proposed method also gives a strongly stable system in this case.

VI. Conclusion

In this paper, a design method of generalized minimum variance control using strong stability rate was given. And

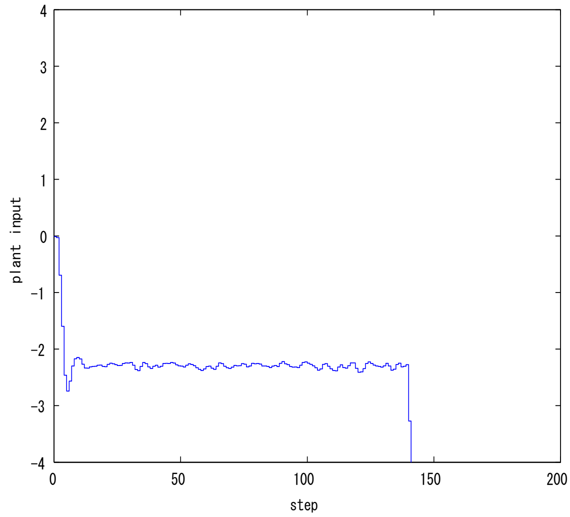

Fig. 2. Conventional GMVC (input)

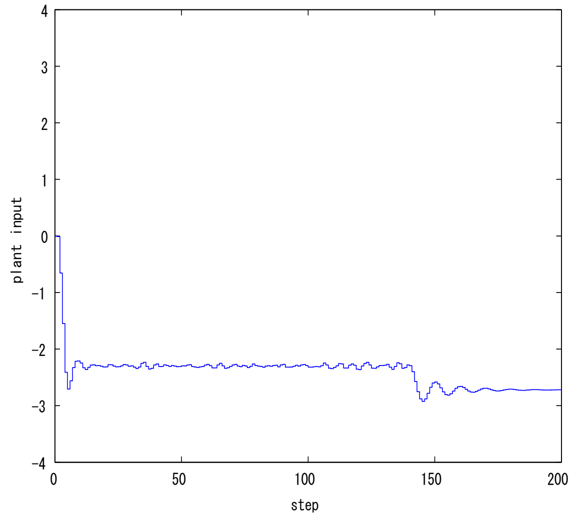

Fig. 4. Proposed method (input)

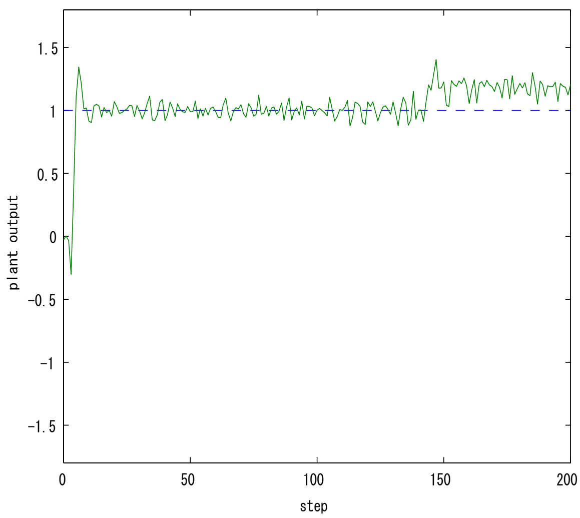

Fig. 3. Proposed method (output)

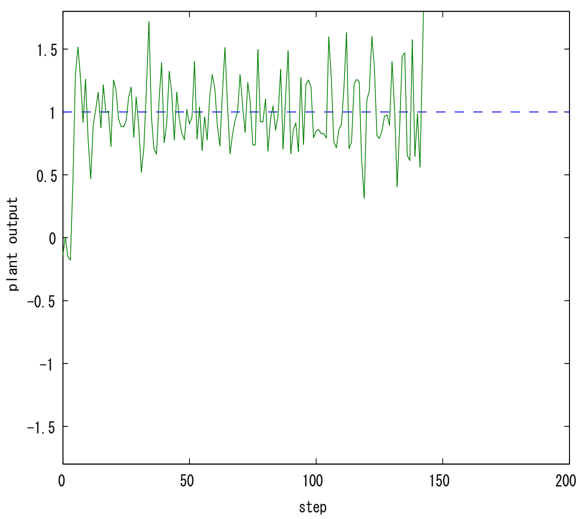

Fig. 5. Conventional GMVC (output)

the numerical example was shown to verify the validity of the proposed method.

As future works, there is an extension to multi-input multi-output systems using the proposed method. Moreover model-free control system through strong stability rate will be considered.

ACKNOWLEDGMENT

This work was supported by JSPS KAKENHI Grant Number 24760337.

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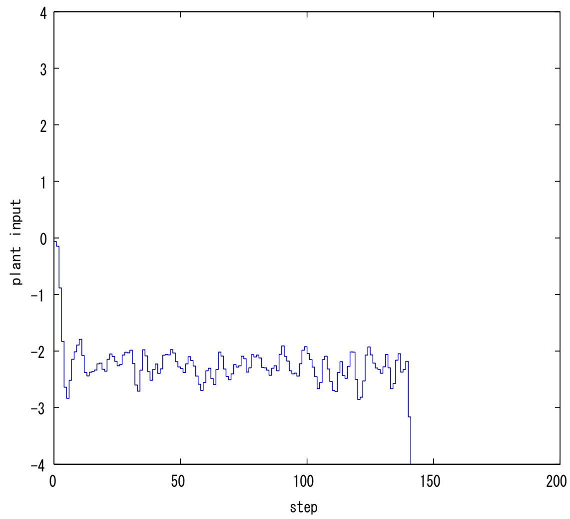

Fig. 6. Conventional GMVC (input)

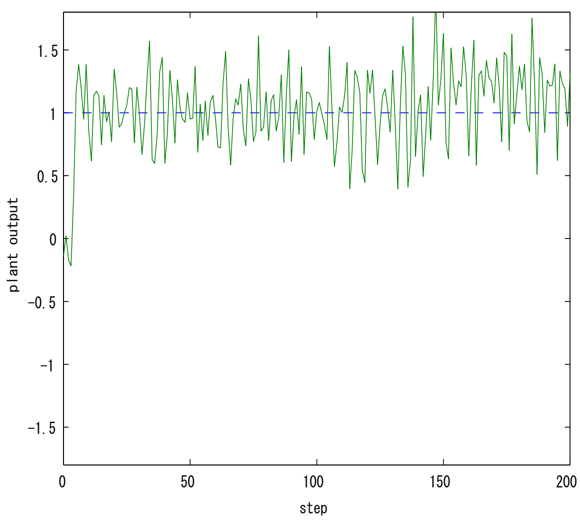

Fig. 7. Proposed method (output)

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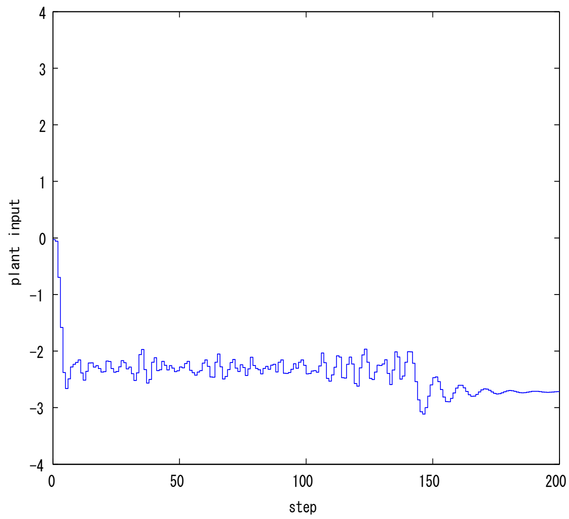

Fig. 8. Proposed method (input)

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