Proposal of Bracing Controller Utilizing Constraint Redundancy and Optimization of Bracing Position

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Abstract: Considering that humans perform handwriting task with small powers by contacting elbow or wrist on a table, we thought that manipulators can save energy and accomplish simultaneously tasks more precisely like human by bracing intermediate links. First this paper discusses equation of motion equation of robot under bracing condition, based on the robot's dynamics with constraint condition including the motor's dynamics. Then, the best elbow-bracing position that minimize the energy consumption is examined simulation on condition that the manipulator has a load on the hand.

Keywords: Constraint motion

1. INTRODUCTION

Humans empirically know that they can write letters correctly with less force with wrist or elbow bracing with tables or desks. This is thought to be an example of humans that can adapt them to environments to have their motions move effective and less energy consuming. We consider how to reduce effect of gravity by using reaction force and perform tasks with high accuracy in less energy by robot bracing with environment[1-6].

In this paper, we discuss about manipulator whose links is being contacted with environment such as a table or floor at plural contacting points. This situation is usually seen in daily behavior of human such as writing a letter as shown in Fig.1.

Fig. 1 Human's writing motion

Redundant manipulator can perform complex tasks by using the redundant degrees of freedom. But, as redundant degree of freedom increase, weight and the number of links rise and then it would be difficult to control manipulators precisely. We consider that it is effective to control robots by mimicking humans' behaviors such as writing letters to solve above problem of gravity influence concerning how to control robots precisely while reducing energy consumed for the controlling. In previous studies, energy consumption and control accuracy (hand accuracy error) are reduced to about one fifth by bracing elbow.

In this report, objects with different weight was attached to the robot's hand, which comprises 4 links and we explore elbow-bracing position that minimizes energy consumption by simulation.

2. MODELING WITH CONSTRAINT CONDITION

2.1 Equation of Motion with Constraint Condition

Here, we describe modeling method of a robot bracing itself by multi-points constraining. $\boldsymbol{q} \in \mathrm{R}^{\mathrm{n}}$ is generalized coordinate and $\boldsymbol{\tau} \in \mathrm{R}^{\mathrm{n}}$ is generalized input. The equation of motion with multi-point constraint condition can be expressed as follows.

$$\begin{aligned}&\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{h}(\boldsymbol{q},\dot{\boldsymbol{q}}) + \boldsymbol{g}(\boldsymbol{q}) + \boldsymbol{D}\dot{\boldsymbol{q}} \\ &\quad = \boldsymbol{\tau} + \left\{ \left(\frac{\partial \boldsymbol{C}}{\partial \boldsymbol{q}^T}\right)^T / \left\| \frac{\partial \boldsymbol{C}}{\partial \boldsymbol{r}^T} \right\| \right\} \boldsymbol{f_n} - \left(\frac{\partial \boldsymbol{r}}{\partial \boldsymbol{q}^T}\right)^T \frac{\dot{\boldsymbol{r}}}{\|\dot{\boldsymbol{r}}\|} \boldsymbol{f_t}\end{aligned} \tag{1}$$

$\boldsymbol{M}$ is $n \times n$ inertia matrix, $\boldsymbol{h}$ and $\boldsymbol{g}$ are $n \times 1$ vectors that indicate the effects from Coriolis force, centrifugal force and gravity. $\boldsymbol{D}$ is $n \times n$ diagonal matrix of coefficients of joints' viscous friction. $\boldsymbol{q}$ is joint angle vector and $\boldsymbol{\tau}$ is input torque vector. $\boldsymbol{f_n}$ represents constraint force vector and $\boldsymbol{f_t}$ does friction force vector.

And, $\boldsymbol{C}$ included in Eq.(1) is an expression of constraint conditions, and Eq.(2) means the number of contacting points and the number of constraint conditions is p. Then constraint condition vector $\boldsymbol{C}$ could be represented as,

$$\begin{aligned}\boldsymbol{C}(\boldsymbol{r}(\boldsymbol{q})) &= [C_1(\boldsymbol{r}_1(\boldsymbol{q})), C_2(\boldsymbol{r}_2(\boldsymbol{q})), \cdots, C_p(\boldsymbol{r}_p(\boldsymbol{q}))]^T \\ &= \boldsymbol{0}\end{aligned} \tag{2}$$

$\boldsymbol{r}_i$ represents the position / orientation of the i-th link of the robot and are subject to kinematics and can be expressed as follows.

$$\boldsymbol{r_i} = \boldsymbol{r_i}(\boldsymbol{q}) \tag{3}$$

$$\dot{\boldsymbol{r_i}} = \boldsymbol{J_i}(\boldsymbol{q})\dot{\boldsymbol{q}} \tag{4}$$

Defining $\boldsymbol{j}_c^T$ and $\boldsymbol{j}_t^T$ as,

$$\left(\frac{\partial C_i}{\partial \boldsymbol{q}^T}\right)^T / \left\| \frac{\partial C_i}{\partial \boldsymbol{r}^T} \right\| = \boldsymbol{j}_{ci}^T \ , \tag{5}$$

$$\left(\frac{\partial \boldsymbol{r}_i}{\partial \boldsymbol{q}^T}\right)^T \frac{\dot{\boldsymbol{r}}_i}{\|\dot{\boldsymbol{r}}_i\|} = \boldsymbol{j}_{ti}^T \ , \tag{6}$$

$\boldsymbol{J}_c^T$, $\boldsymbol{J}_t^T$ can be constituted as,

$$\boldsymbol{J}_c^T = [\boldsymbol{j}_{c1}^T, \ \boldsymbol{j}_{c2}^T, \ \cdots, \ \boldsymbol{j}_{cp}^T], \tag{7}$$

$$\boldsymbol{J}_t^T = [\boldsymbol{j}_{t1}^T, \ \boldsymbol{j}_{t2}^T, \ \cdots, \ \boldsymbol{j}_{tp}^T], \tag{8}$$

$$\boldsymbol{f}_n = [f_{n1}, \ f_{n2}, \ \cdots, \ f_{np}]^T, \tag{9}$$

$$\boldsymbol{f}_t = [f_{t1}, \ f_{t2}, \ \cdots, \ f_{tp}]^T. \tag{10}$$

$\boldsymbol{J}_c^T$, $\boldsymbol{J}_t^T$ are $n \times p$ matrices, $\boldsymbol{f}_n$, $\boldsymbol{f}_t$ are $p \times 1$ vectors. Considering about constraints of the intermediate links, the manipulator's equation of motion can be expressed as,

$$\begin{aligned} &\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{h}(\boldsymbol{q}, \dot{\boldsymbol{q}}) + \boldsymbol{g}(\boldsymbol{q}) + \boldsymbol{D}\dot{\boldsymbol{q}} \\ &= \boldsymbol{\tau} + \sum_{i=1}^{p} (\boldsymbol{j}_{ci}^T f_{ni}) - \sum_{i=1}^{p} (\boldsymbol{j}_{ti}^T f_{ti}) \\ &= \boldsymbol{\tau} + \boldsymbol{J}_c^T \boldsymbol{f}_n - \boldsymbol{J}_t^T \boldsymbol{f}_t \ . \end{aligned} \tag{11}$$

Having Eq.(2) differentiate by time t two times, then we can derive the constraint condition of $\ddot{\boldsymbol{q}}$, $\dot{\boldsymbol{q}}$ and $\boldsymbol{q}$ as,

$$\dot{\boldsymbol{q}}^T \left[\frac{\partial}{\partial \boldsymbol{q}} \left(\frac{\partial \boldsymbol{C}}{\partial \boldsymbol{q}^T} \right) \right] \dot{\boldsymbol{q}} + \left(\frac{\partial \boldsymbol{C}}{\partial \boldsymbol{q}^T} \right) \ddot{\boldsymbol{q}} = 0 \ . \tag{12}$$

To make sure that manipulator's elbow be contacting with the constraint surface all the time, value of $\boldsymbol{q}(t)$ in Eq.(11) has always to satisfy Eq.(2) whenever the time t has any value. For assuring that the solution $\boldsymbol{q}(t)$ of Eq.(11) satisfy the constraint Eq.(2) during any time t, the value of $\ddot{\boldsymbol{q}}$ in Eq.(12) should have the same value with $\ddot{\boldsymbol{q}}$ in Eq.(11). Then value of $\boldsymbol{q}(t)$ in Eq.(11) and Eq.(2) accordingly always have the same value regardless of time. Here, $\boldsymbol{f}_n$ and $\boldsymbol{f}_t$ are related as follows.

$$\boldsymbol{f}_t = \boldsymbol{K}\boldsymbol{f}_n, \ \ \boldsymbol{K} = \mathrm{diag}[\mathrm{K}_1, \mathrm{K}_2, \cdots, \mathrm{K}_\mathrm{p}] \tag{13}$$

Then Eq.(11) can be expressed as,

$$\begin{aligned} &\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{h}(\boldsymbol{q}, \dot{\boldsymbol{q}}) + \boldsymbol{g}(\boldsymbol{q}) + \boldsymbol{D}\dot{\boldsymbol{q}} \\ &\qquad = \boldsymbol{\tau} + (\boldsymbol{J}_c^T - \boldsymbol{J}_t^T \boldsymbol{K})\boldsymbol{f}_n \ . \end{aligned} \tag{14}$$

Eq.(12), (14) can be represented in matrix form as follows,

$$\begin{bmatrix} \boldsymbol{M}(\boldsymbol{q}) & -(\boldsymbol{J}_c^T - \boldsymbol{J}_t^T \boldsymbol{K}) \\ \frac{\partial \boldsymbol{C}}{\partial \boldsymbol{q}^T} & \boldsymbol{0} \end{bmatrix} \begin{bmatrix} \ddot{\boldsymbol{q}} \\ \boldsymbol{f}_n \end{bmatrix} = \begin{bmatrix} \boldsymbol{\tau} - \boldsymbol{h}(\boldsymbol{q}, \dot{\boldsymbol{q}}) - \boldsymbol{g}(\boldsymbol{q}) - \boldsymbol{D}\dot{\boldsymbol{q}} \\ -\dot{\boldsymbol{q}}^T \left[\frac{\partial}{\partial \boldsymbol{q}} \left(\frac{\partial \boldsymbol{C}}{\partial \boldsymbol{q}^T} \right) \right] \dot{\boldsymbol{q}} \end{bmatrix} . \tag{15}$$

2.2 Derivation of reaction force with elbow-bracing

In this section, we state a method how to derive $\boldsymbol{f}_n$. Combining Eq.(12) to (14) and eliminate the $\ddot{\boldsymbol{q}}$, we can get the following equation.

$$\begin{aligned} &\left(\frac{\partial \boldsymbol{C}}{\partial \boldsymbol{q}^\mathrm{T}} \right) \boldsymbol{M}^{-1} \left(\frac{\partial \boldsymbol{C}}{\partial \boldsymbol{q}^\mathrm{T}} \right)^\mathrm{T} \frac{\boldsymbol{f}_n}{\left\| \frac{\partial \boldsymbol{C}}{\partial \boldsymbol{r}^\mathrm{T}} \right\|} \\ &= \left(\frac{\partial \boldsymbol{C}}{\partial \boldsymbol{q}^\mathrm{T}} \right) \boldsymbol{M}^{-1} \left({\boldsymbol{J}_t}^\mathrm{T} \boldsymbol{K} \boldsymbol{f}_n + \boldsymbol{D}\dot{\boldsymbol{q}} + \boldsymbol{h} + \boldsymbol{g} - \boldsymbol{\tau} \right) \\ &\quad - \dot{\boldsymbol{q}}^\mathrm{T} \left[\frac{\partial}{\partial \boldsymbol{q}} \left(\frac{\partial \boldsymbol{C}}{\partial \boldsymbol{q}^\mathrm{T}} \right) \right] \dot{\boldsymbol{q}} \end{aligned} \tag{16}$$

Here, defining $\boldsymbol{M}_c(p \times p)$ and $\boldsymbol{B}(p \times n)$ as follows, we get Eq.(19), when $\boldsymbol{B}$ of $p \times n$ matrices has $\mathrm{rank}(\boldsymbol{B}) = \mathrm{p}$, resulting that $\boldsymbol{B}$ is row full rank.

$$(\partial \boldsymbol{C}/\partial \boldsymbol{q}^T) \boldsymbol{M}^{-1} (\partial \boldsymbol{C}/\partial \boldsymbol{q}^T)^T = \boldsymbol{M}_c \tag{17}$$

$$\left\| \frac{\partial \boldsymbol{C}}{\partial \boldsymbol{r}^\mathrm{T}} \right\| \left(\frac{\partial \boldsymbol{C}}{\partial \boldsymbol{q}^\mathrm{T}} \right) \boldsymbol{M}^{-1} = \boldsymbol{B} \tag{18}$$

$$\begin{aligned} \boldsymbol{M}_c \boldsymbol{f}_n &= \boldsymbol{B} \boldsymbol{J}_t^T \boldsymbol{K} \boldsymbol{f}_n - \boldsymbol{B}\boldsymbol{\tau} + \boldsymbol{B} \{ \boldsymbol{D}\dot{\boldsymbol{q}} + \boldsymbol{h} + \boldsymbol{g} \} \\ &\quad - \left\| \frac{\partial \boldsymbol{C}}{\partial \boldsymbol{r}^\mathrm{T}} \right\| \dot{\boldsymbol{q}}^\mathrm{T} \left[\frac{\partial}{\partial \boldsymbol{q}} \left(\frac{\partial \boldsymbol{C}}{\partial \boldsymbol{q}^\mathrm{T}} \right) \right] \dot{\boldsymbol{q}} \end{aligned} \tag{19}$$

Here, by defining $\boldsymbol{a}$ as,

$$\boldsymbol{a} = \boldsymbol{B} \{ \boldsymbol{D}\dot{\boldsymbol{q}} + \boldsymbol{h} + \boldsymbol{g} \} \tag{20}$$

$$- \left\| \frac{\partial \boldsymbol{C}}{\partial \boldsymbol{r}^\mathrm{T}} \right\| \dot{\boldsymbol{q}}^\mathrm{T} \left[\frac{\partial}{\partial \boldsymbol{q}} \left(\frac{\partial \boldsymbol{C}}{\partial \boldsymbol{q}^\mathrm{T}} \right) \right] \dot{\boldsymbol{q}} \ , \tag{21}$$

Eq.(19) can be simplified as,

$$\boldsymbol{M}_c \boldsymbol{f}_n = \boldsymbol{B} \boldsymbol{J}_t^T \boldsymbol{K} \boldsymbol{f}_n - \boldsymbol{B}\boldsymbol{\tau} + \boldsymbol{a} \ . \tag{22}$$

Moreover, using the following definition of $\boldsymbol{A}(p \times p)$ in Eq. (23), we have Eq.(24).

$$\boldsymbol{A} = \boldsymbol{M}_c - \boldsymbol{B} \boldsymbol{J}_t^T \boldsymbol{K} \tag{23}$$

$$\boldsymbol{A} \boldsymbol{f}_n = \boldsymbol{a} - \boldsymbol{B}\boldsymbol{\tau} \tag{24}$$

Reaction force vector $\boldsymbol{f}_n$ can be obtained from Algebraic equations including input torque $\boldsymbol{\tau}$. Here, combining Eq.(14) to (24), eliminating $\boldsymbol{f}_n$ with the assumption of $|\boldsymbol{A}| \neq 0$ and defining $\boldsymbol{S} = \boldsymbol{J}_c^T - \boldsymbol{J}_t^T \boldsymbol{K}$, we can get the following equation.

$$\begin{aligned} \boldsymbol{M}\ddot{\boldsymbol{q}} + \boldsymbol{h} + \boldsymbol{g} + \boldsymbol{D}\dot{\boldsymbol{q}} &= \boldsymbol{\tau} + \boldsymbol{S}\boldsymbol{A}^{-1}(\boldsymbol{a} - \boldsymbol{B}\boldsymbol{\tau}) \\ &= (\boldsymbol{I} - \boldsymbol{S}\boldsymbol{A}^{-1}\boldsymbol{B})\boldsymbol{\tau} + \boldsymbol{S}\boldsymbol{A}^{-1}\boldsymbol{a} \end{aligned} \tag{25}$$

Eq.(25) means that when we give any torque $\boldsymbol{\tau}$, maipulator always move while satisfying the constraint condition Eq.(2). Then, Fig.2 shows two methods to express constraint motion. One is expressed by Eq.(2) and (14), and the other is done by Eq.(24) and (25). We named this dual aspects of expression on constraint dynamics as "Dual System".

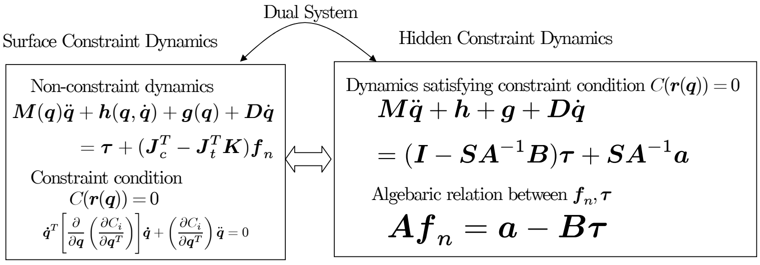

Fig. 2 Dual system of manipulator under constraint condition

2.3 Robot Dynamics including Motor

In this research, we want to evaluate the bracing effects about trajectory tracking accuracy and energy consumption used for countering the gravity force and other effects by bracing the intermediate link. Even though robot is stationary — robot is stopping — the energy is kept to be consuming since motors of joints have to generate torques to maintain the required robot's configuration against gravity influences. When the robot is in motion, other effects of dynamics will be added more to the gravity effect. To evaluate this kind of wasted energy consumption, we included the effects of electronic circuit flowing in servo motors into the equation of motion of the manipulator to represent explicitly that the robot consumes energy even while stopping.

For $i = 1, 2, \cdots, n$, equation of voltage, counter electromotive force, equation of motion and generation of torque can be expressed as follows,

$$v_i(t) = L_i \dot{i}_i + R_i i_i(t) + v_{gi}(t) , \tag{26}$$
$$v_{gi}(t) = K_{Ei}\dot{\theta}_i(t) , \tag{27}$$
$$I_{mi}\ddot{\theta}_i = \tau_{gi}(t) - \tau_{Li}(t) - d_{mi}\dot{\theta}_i , \tag{28}$$
$$\tau_g(t) = K_{Ti} i_i(t) . \tag{29}$$

When v_i is terminal voltage of motor, R_i ; electrical resistance, L_i ; inductance, i_i ; electric current flowing through a circuit, θ_i ; angular displacement of motors, τ_{gi} ; generation of torque, τ_{Li} ; load torque, v_{gi} ; counter electromotive force, I_{mi} ; moment of inertia of motors, K_{Ei} ; constant of counter electromotive force, K_{Ti} ; torque constant, d_{mi} ; coefficient of viscous friction of rotors.

From the relation of magnetic field and the coefficients above, $K_{Ti} = K_{Ei}(= K)$ holds for motors used. Combining Eq.(26) and Eq.(27), and also Eq.(28) and Eq.(29), we derive

$$v_i = L_i \dot{i}_i + R_i i_i + K_i \dot{\theta}_i , \tag{30}$$
$$I_{mi}\ddot{\theta}_i = K_i i_i - \tau_{Li} - d_{mi}\dot{\theta}_i . \tag{31}$$

In the situation with motor and gear whose gear ratio is $k_i > 1$ are installed onto manipulator,

$$\theta_i = k_i q_i , \tag{32}$$
$$\tau_{Li} = \frac{\tau_i}{k_i} . \tag{33}$$

Combining Eq.(30) and Eq.(31) with Eq.(32) and (33), we have

$$L_i \dot{i}_i = v_i - R_i i_i - K_i k_i \dot{q}_i , \tag{34}$$
$$\tau_i = -I_{mi}k_i^2 \ddot{q}_i + K_i k_i i_i - d_{mi}k_i^2 \dot{q}_i . \tag{35}$$

Then using vector and matrix to integrate Eq.(34) and (35) into a combined form,

$$\boldsymbol{L}\dot{\boldsymbol{i}} = \boldsymbol{v} - \boldsymbol{R}\,\boldsymbol{i} - \boldsymbol{K_m}\dot{\boldsymbol{q}} , \tag{36}$$
$$\boldsymbol{\tau} = -\boldsymbol{J_m}\ddot{\boldsymbol{q}} + \boldsymbol{K_m}\boldsymbol{i} - \boldsymbol{D_m}\dot{\boldsymbol{q}} , \tag{37}$$

$$\boldsymbol{v} = [v_1, v_2, \cdots, v_s]^T ,$$
$$\boldsymbol{i} = [i_1, i_2, \cdots, i_s]^T ,$$

and the definitions used in above equations are shown as follow, whose components are positive value.

$$\boldsymbol{L} = diag[L_1, L_2, \cdots, L_s]$$
$$\boldsymbol{R} = diag[R_1, R_2, \cdots, R_s]$$
$$\boldsymbol{K_m} = diag[K_{m1}, K_{m2}, \cdots, K_{ms}]$$
$$\boldsymbol{J_m} = diag[J_{m1}, J_{m2}, \cdots, J_{ms}]$$
$$\boldsymbol{D_m} = diag[D_{m1}, D_{m2}, \cdots, D_{ms}]$$
$$K_{mi} = K_i k_i,\ J_{mi} = I_{mi}k_i^2,\ D_{mi} = d_{mi}k_i^2$$

Substituting Eq.(37) into Eq.(14), we get

$$(\boldsymbol{M}(\boldsymbol{q}) + \boldsymbol{J_m})\ddot{\boldsymbol{q}} + \boldsymbol{h}(\boldsymbol{q},\dot{\boldsymbol{q}}) + \boldsymbol{g}(\boldsymbol{q}) + (\boldsymbol{D} + \boldsymbol{D_m})\dot{\boldsymbol{q}} = \boldsymbol{K_m}\boldsymbol{i} + (\boldsymbol{J}_c^T - \boldsymbol{J}_t^T\boldsymbol{K})\boldsymbol{f_n} \tag{38}$$

Similar to the same relation between Eq. (11) and Eq. (12), the value of $\ddot{\boldsymbol{q}}$ in Eq. (38) have to be identical to the value of $\ddot{\boldsymbol{q}}$ in Eq. (12) representing constrain condition.

2.4 Robot/Motor Equation with Constraint

To make sure that $\ddot{\boldsymbol{q}}$ in Eq.(38) and (12) be identical, constraint force $\boldsymbol{f_n}$ is subordinately decided by simultaneous equation. Eqs.(38) and (12) are transformed as follows

$$(\boldsymbol{M}(\boldsymbol{q}) + \boldsymbol{J_m})\ddot{\boldsymbol{q}} - \boldsymbol{J}_c^T\boldsymbol{f_n} + \boldsymbol{J}_t^T\boldsymbol{K}\boldsymbol{f_n} = \boldsymbol{K_m}\boldsymbol{i} - \boldsymbol{h} - \boldsymbol{g} - (\boldsymbol{D} + \boldsymbol{D_m})\dot{\boldsymbol{q}} \tag{39}$$

$$\left(\frac{\partial \boldsymbol{C}}{\partial \boldsymbol{q}^T}\right)\ddot{\boldsymbol{q}} = -\left[\frac{\partial}{\partial \boldsymbol{q}}\left(\frac{\partial \boldsymbol{C}}{\partial \boldsymbol{q}}\right)\dot{\boldsymbol{q}}\right]\dot{\boldsymbol{q}} = -\dot{\boldsymbol{q}}^T\left[\frac{\partial}{\partial \boldsymbol{q}}\left(\frac{\partial \boldsymbol{C}}{\partial \boldsymbol{q}^T}\right)\right]\dot{\boldsymbol{q}} \tag{40}$$

Then Eqs.(39),(40),(36) can be expressed as follow,

$$\begin{bmatrix} \boldsymbol{M}+\boldsymbol{J_m} & -(\boldsymbol{J}_c^T-\boldsymbol{J}_t^T\boldsymbol{K}) & \boldsymbol{0} \\ \frac{\partial \boldsymbol{C}}{\partial \boldsymbol{q}^T} & 0 & \boldsymbol{0} \\ \boldsymbol{0} & 0 & \boldsymbol{L} \end{bmatrix} \begin{bmatrix} \ddot{\boldsymbol{q}} \\ \boldsymbol{f_n} \\ \dot{\boldsymbol{i}} \end{bmatrix} = \begin{bmatrix} \boldsymbol{K_m i}-\boldsymbol{h}-\boldsymbol{g}-(\boldsymbol{D}+\boldsymbol{D_m})\dot{\boldsymbol{q}} \\ -\dot{\boldsymbol{q}}^T\left[\frac{\partial}{\partial \boldsymbol{q}}(\frac{\partial \boldsymbol{C}}{\partial \boldsymbol{q}^T})\right]\dot{\boldsymbol{q}} \\ \boldsymbol{v}-\boldsymbol{Ri}-\boldsymbol{K_m}\dot{\boldsymbol{q}} \end{bmatrix} \quad (41)$$

Furthermore by redefining as

$$\boldsymbol{M}^* = \begin{bmatrix} \boldsymbol{M}+\boldsymbol{J_m} & -(\boldsymbol{J}_c^T-\boldsymbol{J}_t^T\boldsymbol{K}) & \boldsymbol{0} \\ \frac{\partial \boldsymbol{C}}{\partial \boldsymbol{q}^T} & 0 & 0 \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{L} \end{bmatrix} \quad (42)$$

$$\boldsymbol{b} = \begin{bmatrix} \boldsymbol{K_m i}-\boldsymbol{h}-\boldsymbol{g}-(\boldsymbol{D}+\boldsymbol{D_m})\dot{\boldsymbol{q}} \\ -\dot{\boldsymbol{q}}^T\left[\frac{\partial}{\partial \boldsymbol{q}}(\frac{\partial \boldsymbol{C}}{\partial \boldsymbol{q}^T})\right]\dot{\boldsymbol{q}} \\ \boldsymbol{v}-\boldsymbol{Ri}-\boldsymbol{K_m}\dot{\boldsymbol{q}} \end{bmatrix} \quad (43)$$

Then Eq.(41) can be expressed as,

$$\boldsymbol{M}^* \begin{bmatrix} \ddot{\boldsymbol{q}} \\ \boldsymbol{f_n} \\ \dot{\boldsymbol{i}} \end{bmatrix} = \boldsymbol{b} \quad (44)$$

assuming and calculating $\boldsymbol{M}^*$, then the unknown value of $\ddot{\boldsymbol{q}}$, $\boldsymbol{f_n}$,$\dot{\boldsymbol{i}}$ can be determined based on the above simultaneous equation.

3. CONTROL BY CONSTRAINT REDUNDANCY

control law that control reaction force $\boldsymbol{f_n}$ has the same dimension to constraint condition $\boldsymbol{C}(\boldsymbol{r}(\boldsymbol{q}))$. The torque to achieve the desired contacting force at plural of links $\boldsymbol{f_{nd}}$ can be given by solving Eq.(24) as,

$$\boldsymbol{\tau} = \boldsymbol{B}^+(\boldsymbol{a}-\boldsymbol{A}\boldsymbol{f}_{nd}) + (\boldsymbol{I}-\boldsymbol{B}^+\boldsymbol{B})\boldsymbol{l}\,, \quad (45)$$

$\partial\boldsymbol{C}/\partial\boldsymbol{q}$ is row full rank matrix of $p \times n$. Because $\boldsymbol{M}$ is always regular, $\boldsymbol{B}$ of $p \times n$ matrix has $\mathrm{rank}(\boldsymbol{B}) = \mathrm{p}$ and row full rank. We call $\boldsymbol{B}$ matrix constraint redundant matrix. Since $\boldsymbol{\tau} \in \mathrm{R^n}$, $\boldsymbol{f_{nd}} \in \mathrm{R^p}$ and $n > p$ then there appears a redundancy to achieve $\boldsymbol{f_{nd}}$ by $\boldsymbol{\tau}$. Then remaining redundancy to do some other tasks is $\mathrm{rank}(\boldsymbol{I}-\boldsymbol{B}^+\boldsymbol{B}) = \mathrm{n}-\mathrm{p}$ after controlling reaction force. Therefore, The remaining control input of $\boldsymbol{\tau}$ can be used to track hand target trajectory $\boldsymbol{r}_d$ and for other purposes through arbitrary vector $\boldsymbol{l}$ in Eq.(45).

In simulation of this report, we use one degree of freedom to force control of elbow, one degree of freedom to position control of elbow and two degrees of freedom to position control of hand and control manipulator with four degrees of freedom as shown in Fig.4.

Here, $\boldsymbol{l}$ is used for hand-position and elbow-position control. These are expressed as,

$$\boldsymbol{l} = \boldsymbol{J}_2^T[\boldsymbol{K}_{p2}(y_{2d}-y_2)+\boldsymbol{K}_{d2}(\dot{y}_{2d}-\dot{y}_2)] + \boldsymbol{J}_4^T[\boldsymbol{K}_{p4}(\boldsymbol{r}_{4d}-\boldsymbol{r}_4)+\boldsymbol{K}_{d4}(\dot{\boldsymbol{r}}_{4d}-\dot{\boldsymbol{r}}_4)]\,, \quad (46)$$

where, $\boldsymbol{J}_4$ is hand's Jacobian matrix, $\boldsymbol{J}_2$ is elbow's Jacobian matrix, $\boldsymbol{K}_{p4}$ and $\boldsymbol{K}_{p2}$ is proportional gain, $\boldsymbol{K}_{d4}$, $\boldsymbol{K}_{d2}$ is rate gain, $\boldsymbol{r}_{4d} = [y_{4d}, z_{4d}]^T$ is hand's target position, y_{2d} is elbow's target position, $\boldsymbol{r}_n = [y_4, z_4]^T$ is hand's real position and y_2 is elbow's real position.

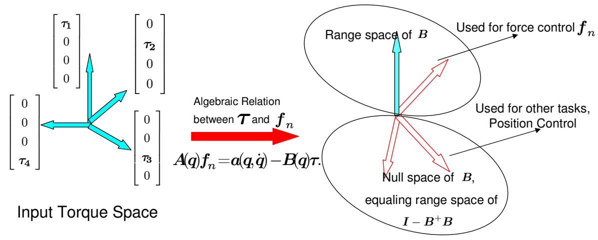

Fig. 3 Algebraic relation

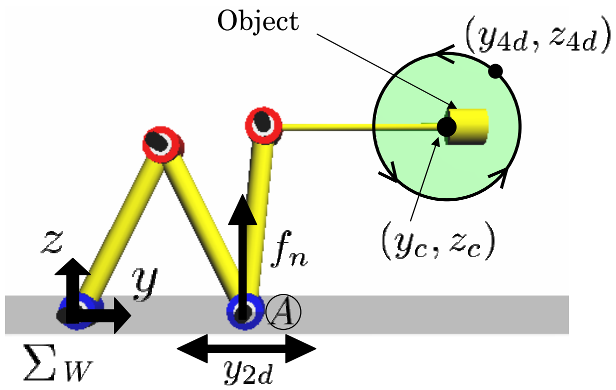

Fig. 4 Simulation model

Figure.3 expresses relation of space between input torque and task space from the viewpoint of range space of $\boldsymbol{B}$ —used for controlling $\boldsymbol{f_{nd}}$— and null space $\boldsymbol{f_{nd}}$. i.e., $\boldsymbol{I}-\boldsymbol{B}^+\boldsymbol{B}$ used for controlling elbow's position in one dimension and hand's position in two dimensions.

We suppose that $p \times n$ matrix $\boldsymbol{B}(\boldsymbol{q})$ is row full rank. When we combine Eq. (24) to (45) and eliminate the $\boldsymbol{\tau}$ to confirm $\boldsymbol{\tau}$ in Eq.(45) can guarantee to generate $\boldsymbol{f_{nd}}$, we can get the following relation,

$$\begin{aligned} \boldsymbol{A}\boldsymbol{f}_n &= \boldsymbol{a}-\boldsymbol{B}\{\boldsymbol{B}^+(\boldsymbol{a}-\boldsymbol{A}\boldsymbol{f_n})+(\boldsymbol{I}-\boldsymbol{B}^+\boldsymbol{B}\boldsymbol{l})\} \\ &= \boldsymbol{A}\boldsymbol{f}_{nd}\,. \end{aligned} \quad (47)$$

When $p \times p$ matrix $\boldsymbol{A}$ has inverse matrix, we have $\boldsymbol{f}_n = \boldsymbol{f}_{nd}$.

Referencing the controller in Eq.(45), another controller with voltage input can be defined as,

$$\boldsymbol{v} = \boldsymbol{K}_v[\boldsymbol{B}^+(\boldsymbol{a}-\boldsymbol{A}\boldsymbol{f}_{nd})+(\boldsymbol{I}-\boldsymbol{B}^+\boldsymbol{B})\boldsymbol{l}]\,, \quad (48)$$

where, $\boldsymbol{K}_v$ is a coefficient that transform torque into voltage.

4. SIMULATION

In this chapter, we state optimization of elbow-bracing position. Links energy consumption $E_i(T)$ used in motor's circuit of mechanical motion can be expressed as follows.

$$E_i(T) = \int_0^T v_i(t)i_i(t)dt \quad (49)$$

Summation of all energy consumption $E_{sum}(T)$ can be expressed as follows.

$$E_{sum}(T) = \sum_{i=1}^{4} E_i(T) \tag{50}$$

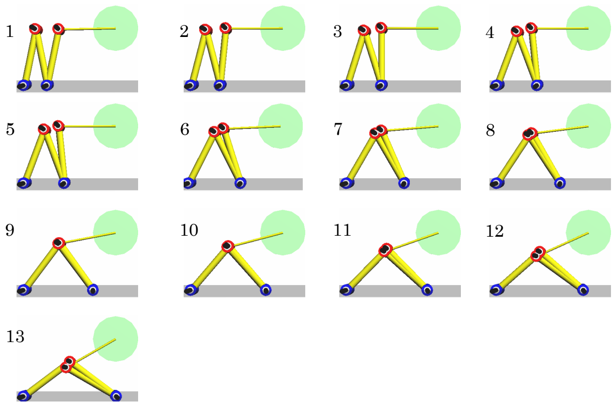

Fig. 5 Elbow-bracing position in simulation

Simulation model that is used in this chapter is shown in Fig.4. We assume that objects was attached to 4 links manipulator and simulated hand-trajectory tracking. Simulation conditions have been set as : each link's weight is $m_i = 1.0$[kg], length is $l_i = 0.5$[m], viscous friction coefficient of joint is $D_i = 2.9$[N $\cdot$ m $\cdot$ s/rad], torque constant is $K_i = 0.2$[N $\cdot$ m/A], resistance is $R_i = 0.6[\Omega]$, inductance is $L_i = 0.17$[H], inertia moment of motor is $Im_i = 1.64{\times}10^{-4}$[kg $\cdot$ m^2], reduction ratio is $k_i = 3.0$, viscous friction coefficient of reducer is $dm_i = 0.1$[N $\cdot$ m $\cdot$ s/rad], proportional gain of hand is $k_{p4} = 300$[N/m], rate gain of hand is $k_{d4} = 50$[N $\cdot$ s/m], proportional gain of elbow is $k_{p2} = 200$[N/m] and rate gain of elbow is $k_{d2} = 40$[N $\cdot$ s/m].

Hand-target trajectory is set as follows,

$$y_d = 0.2\cos\frac{2\pi}{T}t + y_c\ , \tag{51}$$

$$z_d = 0.2\sin\frac{2\pi}{T}t + z_c\ . \tag{52}$$

Center coordinates of target trajectory are $(y_c, z_c) = (0.8, 0.5), (0.9, 0.5), (1.0, 0.5)$. The weight of object that was attached to the hand varied from 0.2 to 1.2 [kg] by 0.2 [kg]. We change elbow-bracing position in each case as Fig.5 and simulate hand-trajectory tracking. Graphs of energy consumption to elbow-bracing position in the simulation are shown in Fig.6-11. Here, elbow-bracing position is distance from origin of work coordinate system Σ_W to A point in Fig.4. From Fig.6-11, each optimum elbow-bracing positions are $0.4[\mathrm{m}], 0.25[\mathrm{m}], 0.2[\mathrm{m}], 0.15[\mathrm{m}], 0.15[\mathrm{m}], 0.1[\mathrm{m}]$ in negative direction of the y-axis from center of target trajectory. Energy consumption is minimized in these elbow-bracing positions. Then, we found to be a parabolic convex downward in all graphs. From this, we consider to be able to explore the optimum elbow-bracing position while manipulator works without knowing it and the form of graph.

5. CONCLUSION

In this paper, we propose bracing elbow, model constraint motion and explore optimum elbow-bracing position by simulation after designing controller. It can be seen that there is the optimum elbow-bracing position that determined depending on the weight of object grasping. In the future, we are going to do optimum control of elbow-bracing position in real time from the results obtained by the simulation.

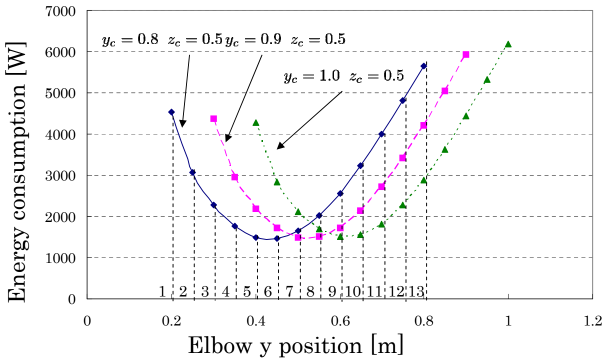

Fig. 6 Evaluation of energy consumption(M=0.0)

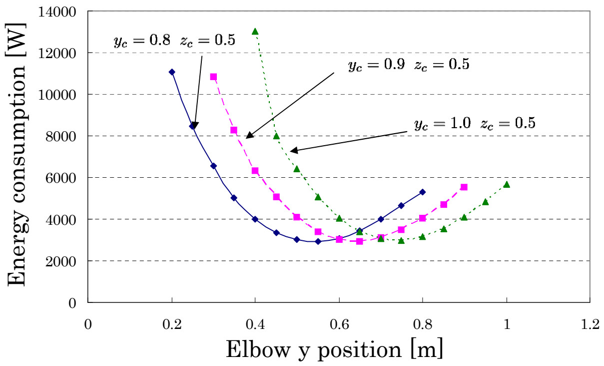

Fig. 7 Evaluation of energy consumption(M=0.4)

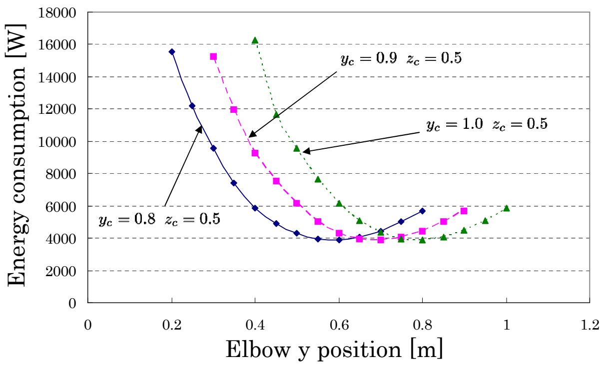

Fig. 8 Evaluation of energy consumption(M=0.6)

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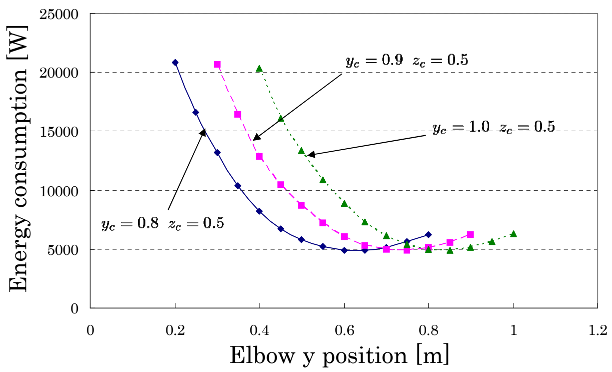

Fig. 9 Evaluation of energy consumption(M=0.8)

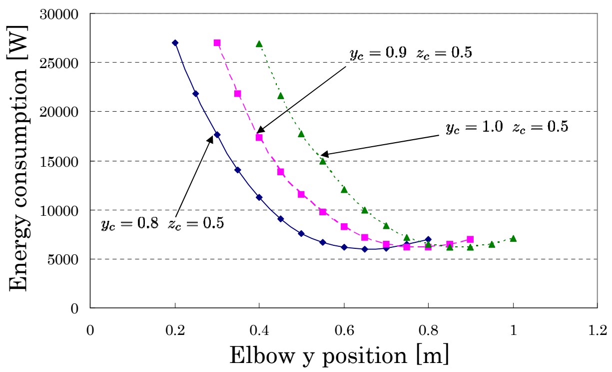

Fig. 10 Evaluation of energy consumption(M=1.0)

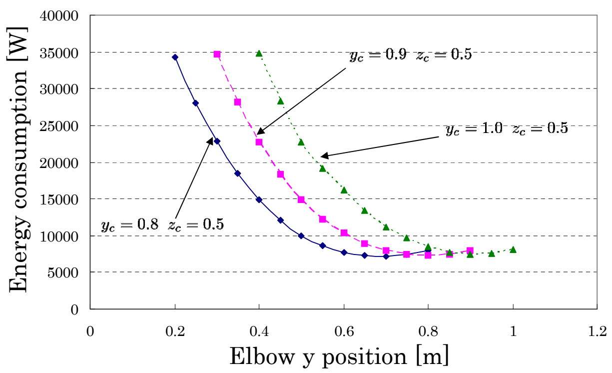

Fig. 11 Evaluation of energy consumption(M=1.2)

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