Constraint-combined Force/Position Hybrid Control
Method with Lyapunov Stability

Fujia Yu¹, Mamoru Minami¹, Tomohide Maeba¹ and Akira Yanou¹
¹ Okayama University, Tsushimanaka3-1-1 Okayama JAPAN
({yufujia, minami, maeba, yanou}@suri.sys.okayama-u.ac.jp)

Abstract: Based on the analysis of the interaction between a manipulator’s hand and a working object, a model representing the constrained dynamics of the robot is first discussed. In this paper, we design a new compensation of the system based on the constrained system and prove its convergence in a new way with Lyapunov method, in this way we focus on the motion of the manipulator because the output force is a function of the input torque directly which is not affected by time.

Keywords: Constraint-combined, Gridding robot, Lyapunov Stability.

1. INTRODUCTION

Many researches have discussed on the constraint-combined force/position hybrid control method, to ensure the stabilities of the constrained motion, those force and position control methods have utilized Lyapunov’s stability analysis under the inverse dynamic compensation [6]-[8]. But most of these proofs are trying to verify their stability under the inverse dynamic compensation and prove its convergence in a new way with Lyapunov method, in this way we focus on the motion of the manipulator because the output force is a function of the input torque directly which is not affected by time.

Those former classical robot controlling approaches can be classified into two broad categories[8]: impedance control and hybrid (force/position) control. In impedance control, a prescribed dynamic relation is sought to be maintained between the robot end-effector’s force exerting to a object constraining the end-effector and position displacement toward the direction vertical to the object’s surface [12]. In hybrid control, the end-effector’s force is explicitly controlled in selected directions and the end-effector’s position is controlled in the remaining (complementary) directions [1].

The hybrid control approaches can be further classified into three main categories: 1) explicit (model based) hybrid control of rigid robots in elastic contact with a compliant environment, e.g., [15], in which the end-effector force is controlled indirectly by modifying the reference trajectory given into an inner loop joint position/velocity controller based on the sensed force error; and 3) explicit (model based) hybrid control of rigid robots in hard contact with a rigid environment, e.g., [1],[3].

According to these classified categories, our force/position control approach named as Constraint-Combined Control, which will be introduced in detailed later should be classified into category 3). In all the former force/position controlling methods of hybrid control of category 1) and 2), the contact surface’s compliant characteristics must be properly taken into account since it will affect force control procedure. As a result, when contact constraint force is analyzed, process the end-effector contacting with constraint surface is being expressed as a motion equation with spring model, which is a differential function with time-varying.

Eq.(1), which has been pointed out by Hemami [16] in the analysis of biped walking robot, denotes also algebraic relation between the input torque \( \tau \) of the robot and exerting force to the working object \( F_n \), when robot’s end-effector being in touch with a surface in 3-D space:

\[
F_n = a(x_1, x_2) - A(x_1) \tau ,
\]

where, \( x_1 \) and \( x_2 \) are state variables. \( a(x_1, x_2) \) and \( A(x_1) \) are scalar function and vector one defined in following section.

In this paper, position and force control performances of our new controller [17] are confirmed by grinding experiments, especially on the view point that the force control space and the position control space are divided into orthogonal spaces being complement each other, that is, force space is defined by range space of \( A \) and the other is null space of \( A \), \((I - A^+ A)\).

2. ANALYSIS OF GRINDING TASK

There are four kinds of grinding processes in common use, called respectively vertical surface grinding, horizontal surface grinding, internal grinding and cylindrical
grinding. A grinding machine usually can only perform one or two of these processes because of kinematic limitation. However, all of the four kinds of tasks can be finished by a single robot manipulator for its dexterity in movement. To do so, the grinding wheel has to contact with the workpiece. A set of contacting surfaces, especially the surfaces being machined, will form constraints to the motions of the grinding wheel. As for vertical surface grinding operation shown in Fig. 1 (a), the grinding wheel in contact with a surface of the work-piece is not free to move through that surface, which forms a position constraint. And also, the wheel cannot freely apply arbitrary force tangent to the surface in case of no disturbing force like friction existing, which forms a set of force constraint. Situations of constraints for other kinds of grinding tasks are shown in Fig. 1 (b), (c) and (d).

Generally speaking, the grinding power is related to the metal removal rate (weight of metal being removed within unit time), which is determined by the depth of cut, the width of cut, the linear velocity of the grinding wheel, the feed rate and so on. There are many empirical formulae available for the determination of grinding power, and the desired force trajectory can then be planned according to the power. The normal grinding force $F_n$ is exerted in the perpendicular direction of the surface. It is a significant factor that affects ground accuracy and surface roughness of workpiece. The value of it is also related to the grinding power or directly to the tangential grinding force as

$$F_t = K_t F_n,$$

(2)

where, $K_t$ is an empirical coefficient, $F_t$ is the tangential grinding force. The axial grinding force $F_n$ is proportional with the feed rate, and is much smaller than the former force.

Eq. (2) is based on the situation that position of the grinding cutter is controlled like currently used machining center. But when a robot is used for the grinding task, the exerting force to the object and the position of the grinding cutter should be controlled simultaneously. The $F_n$ is generally determined by the constrained situation, and it is not suitable to apply Eq. (2) to grinding motion by the robots.

For grinding task, the normal force and tangential velocity are the most important two factors. To improve grinding quality, it is usually desired that the normal force is constant while the velocity is also constant in the middle term of a grinding stroke.

3. MODELLING

3.1 Constrained Dynamic Systems

Hemami and Wyman have addressed the issue of control of a moving robot according to constraint condition and examined the problem of the control of the biped locomotion constrained in the frontal plane. Their purpose was to control the position coordinates of the biped locomotion rather than generalized forces of constrained dynamic equation involved the item of generalized forces of constraints. And the constrained force is used as a determining condition to change the dynamic model from constrained motion to free motion of the legs. In this paper, the grinding manipulator which is shown in Fig. 2 whose end-point is in contact with the constrained surface, is modeled according Eq. (7) with Lagrangian equations of motion in term of the constraint forces, referring to what
Hemami and Arimoto have done:

\[
\begin{align*}
\frac{d}{dt} \left[ \frac{\partial L}{\partial \dot{q}^i} \right] - \frac{\partial L}{\partial q^i} &= \tau + (\frac{\partial C}{\partial q^i})^T/l \| \frac{\partial C}{\partial \dot{q}} \| F_n - (\frac{\partial r}{\partial \dot{q}}) \dot{r} / \| r \| F_t \\
&= \tau + (\frac{\partial r}{\partial \dot{q}})((\frac{\partial C}{\partial \dot{r}})/l) \frac{\partial C}{\partial q^i} F_n - \frac{\dot{r}}{\| r \|} F_t \quad (3)
\end{align*}
\]

\( r \) is the \( l \) position vector of the hand and can be expressed as a kinematic equation,

\[ r = r(q). \quad (4) \]

\( L \) is the Lagrangian function, \( q \) is \( l \geq 2 \) generalized coordinates, \( \tau \) is \( l \) generalized torque inputs. The discussing robot system does not have kinematic redundancy. \( C \) is a scalar function of the constraint, and is expressed as an equation of constraints

\[ C(r(q)) = 0, \quad (5) \]

\( F_n \) is the constrained force associated with \( C \) and \( F_t \) is the tangential disturbance force.

It is easy to see that here \( F_n \), \( F_t \) express the value of the pressure and the friction respectively, while \( \delta C/\delta \dot{r} \) and \( \dot{r} / \| r \| \) express the direction of the pressure and the friction, because the friction is always vertical to the pressure, we can get

\[ \left( \frac{\partial C}{\partial \dot{r}} / \| \frac{\partial C}{\partial \dot{q}} \| \right)^T \cdot \frac{\dot{r}}{\| r \|} = 0 \quad (6) \]

we define \( J_c \) and \( J_r \) as

\[ J_c = \frac{\partial C}{\partial \dot{q}} / \| \frac{\partial C}{\partial \dot{r}} \| = \left[ \frac{\partial C}{\partial q^i} / \| \frac{\partial C}{\partial r^j} \| \right], \quad J_r = \frac{\partial r}{\partial \dot{q}} \quad J_r = \frac{\partial C}{\partial \dot{r}} / \| \dot{r} \|, \quad (3) \]

can be rewritten into

\[ \frac{d}{dt} \left[ \frac{\partial L}{\partial q^i} \right] - \frac{\partial L}{\partial q^i} = \tau + J_c^T(q) F_n - J_r^T(q) F_t \quad (7) \]

Eq. (7) can be derived to

\[ M \ddot{q} + H(q, \dot{q}) + G = \tau + J_c^T(q) F_n - J_r^T(q) F_t, \quad (8) \]

where \( M \) is an \( l \times l \) matrix, \( H \) and \( G \) are \( l \) vectors. Because

\[ \frac{\partial C}{\partial \dot{q}^i} \ddot{q} = -\frac{\partial C}{\partial \dot{q}^i} \dot{q} \\dot{q} = -\dot{q}^T \left[ \frac{\partial C}{\partial \dot{q}^i} \right] \dot{q} \quad (9) \]

And equation (8) can be rewritten as follows:

\[ \begin{bmatrix} M & -J_c^T \\ \frac{\partial C}{\partial q^i} & 0 \end{bmatrix} \begin{bmatrix} \ddot{q} \\ F_n \end{bmatrix} = \left[ \begin{array}{c} \tau - H(q, \dot{q}) - J_r^T(q) F_t \\ -\dot{q}^T \left[ \frac{\partial C}{\partial \dot{q}} \right] \dot{q} \end{array} \right] \quad (10) \]

here we define

\[ \tilde{M} = \begin{bmatrix} M & -J_c^T \\ \frac{\partial C}{\partial q^i} & 0 \end{bmatrix} \quad (11) \]

in equation (10) if and only if \( \tilde{M} \) is full rank matrix we can control the manipulator to the desired pose. Here we express \( \tilde{M} \) in a simple form:

\[ \tilde{M} = \begin{bmatrix} M & -lA^T \\ A & 0 \end{bmatrix} = \begin{bmatrix} M & -J_c^T \\ \frac{\partial C}{\partial \dot{q}^i} & 0 \end{bmatrix} \quad (12) \]

here \( l = 1 / \| \frac{\partial C}{\partial \dot{r}^i} \| > 0 \), because \( M \) is positive definite, there exist a orthogonal matrix \( V \) which can makes

\[ V M V^T = \begin{bmatrix} \lambda_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \lambda_n \end{bmatrix} (\lambda_1 \cdots \lambda_n > 0) \quad (13) \]

so

\[ \det(\tilde{M}) = \det(M^{-1}) \det(AMlA^T) = l \cdot \det(M^{-1}) \det(AMlV^T V M V^T V A^T) \]

define

\[ A V^T = [a_1, \ldots, a_n] \quad (15) \]

so

\[ \det(\tilde{M}) = l \cdot \det(M) \sum_{i=1}^{n} (\lambda_1 a_i^2) > 0 \quad (16) \]

which means that \( \tilde{M} \) is reversible.

The state variable \( x \) is constructed by adjoining \( q \) and \( \dot{q} \):

\[ x = (x_c^T, x_c^T)^T = (q^T, \dot{q}^T)^T. \]

The state-space equation of the system are

\[ \begin{align*}
\dot{x}_1 &= x_2,
\dot{x}_2 &= -M^{-1}(H(x_1, x_2) + G(x_1)) + M^{-1}(\tau + J_c^T(x_1) F_n - J_r^T(x_1) F_t) \\
& \text{or in the compact form}
\end{align*} \quad (17) \]

\[ \dot{x} = F(x, \tau, F_n, F_t), \quad (18) \]

Using the inverted form of combination from Eq. (5) and Eq. (18) (this part had been detailedly introduced in [19] by us), \( F_n \) can be expressed as

\[ F_n = F_n(x, \tau, F_t), \quad (19) \]

or in a more detailed form

\[ F_n = \left( \frac{\partial C}{\partial q^i} \right) M^{-1} \left( \frac{\partial C}{\partial \dot{q}^i} \right)^{-1} \| \frac{\partial C}{\partial \dot{r}^i} \| \]

\[ = -\left[ \frac{\partial C}{\partial q^i} \right] \frac{\partial C}{\partial \dot{r}^i} M^{-1} (H(q, \dot{q}) + G(q) + J_r^T F_t) \}

\[ \\triangleq a(x_1, x_2) + A(x_1) J_c^T F_t - A(x_1) \tau, \quad (20) \]

where, \( a(x_1, x_2) \) is a scalar representing the first term in the expression of \( F_n \), and \( A(x_1) \) is an \( l \) vector to represent the coefficient vector of \( \tau \) in the same expression.

Eq. (18) and Eq. (19) compose a constrained system that can be controlled, if \( F_n = 0 \), describing the unconstrained motion of the system.
Substituting Eq. (20) into Eq. (17), the state equation of the system including the constrained force (as \( F_n > 0 \)) can be rewritten as

\[
\dot{x}_1 = x_2,
\]

\[
x_2 = -M^{-1}[H(x_1, x_2) + G(x_1) - J_e^T(x_1)a(x_1, x_2)] + M^{-1}[(I - J_e^T A)\tau + (J_e^T A - I)J^T F_t].
\]

Solutions of these dynamic equation always satisfy the constrained condition (5).

4. FORCE AND POSITION CONTROLLER WITH LYAPUNOV STABILITY

4.1 Preparation for the proof

From Lagrange equation we can get the dynamics equation of the manipulator

\[
M(q)\ddot{q} + \dot{M}(q)\dot{q} - \frac{1}{2} \frac{\partial}{\partial q}(q^T M(q)\dot{q}) + G(q) = \tau
\]  

(22)

here \( M(q) \) is the inertial moment matrix, \( q \) express the angle of the joints, \( q \) is the gravity term. The energy of a manipulator system is:

\[
E = K + U = \frac{1}{2} q^T M(q)q + U(q)
\]  

(23)

(24)

here, \( E \) is the whole energy of the system, \( K \) is kinetic energy and \( U \) is the gravity potential energy, so

\[
\dot{E} = \frac{1}{2} q^T M(q)\dot{q} + \frac{1}{2} \dot{q}^T M(q)(\ddot{q}) + \frac{1}{2} \ddot{q}^T M(q)\dot{q} - \frac{\partial U}{\partial q} \dot{q}
\]  

(25)

(26)

from (22) and (26) we get

\[
q^T (M(q)\ddot{q} + \frac{1}{2} \dot{M}(q)\dot{q} + \frac{1}{2} q^T M(q)\dot{q})
\]  

(27)

\[
= -q^T \left( \frac{1}{2} \dot{M}(q)\dot{q} - \frac{1}{2} \frac{\partial}{\partial q^T}(q^T M(q)\dot{q}) \right)
\]  

(28)

\[
= -\frac{1}{2} q^T \left( \dot{M}(q)\dot{q} - \frac{\partial}{\partial q^T}(q^T M(q)\dot{q}) \right)
\]  

(29)

set

\[
S(q, \dot{q}) = \frac{1}{2} \left( \dot{M}(q) - \frac{\partial}{\partial q^T}(q^T M(q)\dot{q}) \right)
\]  

(30)

so (29) can be simplified to

\[
q^T (M(q)\ddot{q} + \frac{1}{2} \dot{M}(q)\dot{q} + G(q)) = -q^T S(q, \dot{q})\dot{q}
\]  

(31)

It is easy to see that \( S(q, \dot{q}) \) is a skew-symmetrical matrix, and the manipulator dynamics equation (22) can be written as

\[
M(q)\ddot{q} + \frac{1}{2} \dot{M}(q)\dot{q} + S(q, \dot{q})\dot{q} + G(q) = \tau.
\]  

(32)

4.2 Force and position controller with Lyapunov Stability

We make the input torque as:

\[
\tau = -A^+ (x_1) [F_{nd} - a(x_1, x_2) - A(x_1)J^T F_t]
\]

\[
- (I - A^+ (x_1) A(x_1))k
\]  

(33)

here we define \( N = I - A^+ A \), and where \( J \) is a \( l \times l \) identity matrix, \( F_{nd} \) is the desired constrained forces, \( A(x_1) \) is defined in Eq. (20) and \( A^T(x_1) \) is the pseudoinverse matrix of \( A(x_1) \), \( a(x_1, x_2) \) is also defined in Eq. (20) and \( k \) is an arbitrary vector which is defined as

\[
k = J^T F_n - J^T F_t - c[p_1(x_1, x_2) + k_2r_x]
\]  

(34)

because \( V^T N \neq 0 \), there always exist a vector \( c \) make \( V^T N c \neq 0 \), here \( c \) is a arbitrary vector which satisfies \( V^T N c = 0 \). The position of the end-effector can be expressed as \( \tau = [r_x, r_y, r_z] \) so,

\[
\dot{q} = V\dot{r}_x
\]  

(35)

here, \( V \) is a \( 3 \times 3 \) vector, differentiate both side of (35) by time

\[
\ddot{q} = \dot{V}\dot{r}_x + V\ddot{r}_x
\]  

(36)

here we do not consider the gravity and friction so the dynamics equation of the system can be simplified to:

\[
M(q)\ddot{q} + H\dot{q} = \tau + J_e^T F_n - J_e^T (q)F_t
\]  

(37)

(38)

premultiply \( V^T N \) on both side we can get

\[
V^T N M(q)\dot{r}_x + V^T N (M(q))\dot{r}_x
\]

\[
+ V^T N H\dot{r}_x = V^T N \tau + V^T N J_e^T F_n - V^T N J_e^T (q)F_t
\]  

(39)

Take (33) into (39) we can get

\[
V^T N M(q)\dot{r}_x + V^T N M(q)\dot{r}_x + V^T N H\dot{r}_x
\]

\[
= V^T N k + V^T N J_e^T F_n - V^T N J_e^T (q)F_t
\]

\[
= V^T N c(-k_0r_x - k_1p_1(x_1, x_2) + k_2p_2(x_1, x_2))
\]

\[
= -k_0\dot{r}_x - k_1\dot{p}_1(x_1, x_2) + k_2\dot{p}_2(x_1, x_2)
\]  

(40)

(41)

(42)

\( H \) can be divided into \( \frac{1}{2} \dot{M}(q) + S \) here \( S \) is an antisymmetric matrix, so (40) can be written as:

\[
V^T N M(q)\dot{r}_x + V^T N M(q)\dot{r}_x + V^T N H\dot{r}_x
\]

\[
+ V^T N \left( \frac{1}{2} \dot{M}(q) + S \right)\dot{r}_x + k_0\dot{r}_x + k_1\dot{p}_1(x_1, x_2) + k_2\dot{p}_2(x_1, x_2)
\]

\[
= \ 0
\]  

(43)

multiple \( \dot{r}_x \) on the both side of (43) we can get:

\[
V^T N M(q)\dot{r}_x + V^T N M(q)\dot{r}_x + V^T N (M(q))\dot{r}_x
\]

\[
+ V^T N \left( \frac{1}{2} \dot{M}(q) + S \right)\dot{r}_x + k_0\dot{r}_x + k_1\dot{p}_1(x_1, x_2) + k_2\dot{p}_2(x_1, x_2)
\]

\[
= \ 0
\]  

(44)
set Lyapunov argument as:
\[
V = \frac{1}{2} V^T N M(q) V \dot{r}_x^2 + \int_0^t \frac{1}{2} V^T A^+ A M(q) V \dot{r}_x^2 dt \\
+ \frac{1}{2} k_p (r_x - r_{xd})^2 \geq 0
\] (45)
so
\[
\dot{V} = V^T N M(q) V \ddot{r}_x \dot{r}_x + V^T N M(q) \dot{V} \dot{r}_x \\
+ V^T N \frac{1}{2} M(q) V \dot{r}_x^2 + k_p (r_x - r_{xd}) \dot{r}
\] (46)
from (44), (46) can be transformed to
\[
\dot{V} = -V^T S V \ddot{r}_x^2 - k_d \dot{r}_x^2 \\
= -k_d \dot{r}_x^2 \leq 0
\] (47)
because \( S \) is a skew symmetrical matrix. From (46) we can see only when \( \dot{r}_x = 0, V = 0 \), in this case we assume that \( r_x \neq r_{xd} \) so from (33) we can know that \( \tau \neq 0 \) this conflict to \( \dot{r}_x = 0 \) so we can say that if and only if \( r_x = r_{xd}, \dot{r}_x = 0 \) and \( V = 0 \), and then get the conclusion:
\[
\lim_{t \to \infty} r_x = r_{xd}
\] (48)
because the constraint system satisfies that \( C(r(q)) = 0 \),
\[
\lim_{t \to \infty} r = \lim_{t \to \infty} r_{xd}
\] (49)
Because (20) is a function does not affected by time, when we substitute (34) into (20), we can get
\[
F_n = F_{nd}
\] (50)
here (50) does not conclude the variable of time which means that, the output force always equals the desired one.

5. CONCLUSIONS

In this paper we designed a constraint-combined force/position controller for the continuous shape-grinding system, and prove the convergence of the controller in a new way by Lyapunov method, the output force of the system can always equel to the desired once. At last we did some simulations to verify the convergence of the controller.


