Modelling and Control of Hyper-Redundancy Mobile Manipulator
Bracing Multi-Elbows for High Accuracy / Low-Energy Consumption

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Abstract—This paper presents a practical dynamical model of hyper-redundant mobile manipulator whose plural intermediate links are being braced with environment. To overcome the conflict between the required high-redundancy for dexterous manipulation and heavy weight stemming from the high redundancy, we discuss a realizability whether the contacting and bracing motion of intermediate links with environment may simultaneously prevent from overturning and reduce energy-consumption and raise hand’s trajectory tracking accuracy, inspired by human’s handwriting motion with the elbow or wrist contacting to a table. Finally the simulation result shows that less-energy consumption and high trajectory tracking accuracy are achieved by conventional PD controller, compared with non-contacting and non-basement-moving manipulator condition.

I. INTRODUCTION

Hindrances interfering realistic and practical utilization of hyper-redundant manipulator is thought to be the facts that the higher redundant degrees make the weight of the structure heavier, resulting in some difficulties in the controlling, accuracy and stability including whether the hyper-redundant mobile manipulator will overturn or not. For solving this problem we obtain some inspirations about effective motion control strategies by observing human’s handwriting motion. Writing a character on a paper with contacting one’s elbow as shown Fig. 1(a), which is one of the examples of human’s skillful behavior thought to be exploiting the contact constraint of the elbow with the table for reducing inputting energy by counteracting the gravity effects with reaction forces.

On the other hand, hyper-redundant manipulator had been researched intensively and those efforts had been introduced by Chirikjian and Burdick[1] more than ten years ago, where the structure of the discussed hyper-redundant manipulator could not move in 3D space but restricted in the 2D space on a surface of table. Though considerable researches have discussed how to utilize the redundancy [2]-[5], for example avoiding obstacles [6]-[9] or optimizing the configuration concerning practical criteria [10],[11], etc.. However it seems that the merits of the hyper-redundancy has not been utilized enough effectively and practically. Here we think the reason that higher redundant degree causes heavier weight of structure, then more easily its end effector falls down by gravity influence, which has the control precision of the end-effector getting worse.

Therefore up to now there are many researches to discuss the effectiveness and accuracy of the hyper-redundant manipulator with constraint due to contact with the environment. West and Asada[12] presented a general kinematics contact model for the design of hybrid position/force controllers for constrained manipulator. And then a multi-contact kinematic model to control manipulator’s contact motion was also presented in[13],[14], in which they assumed the contact environment as a spring model. However actually the contact environment is almost rigid even if it can happen changed, which must be also offered a rather large force on a premise that the contact environment can not be broken. Moreover the contact point of manipulator will shake with respect to the contact environment due to the unstable of the contact force. So this spring contact environment model is somewhat not the best correct approach to represent contacting nature. Contrarily in this paper we will discuss a model without contacting deformation of environment.

In this research, we propose a new dynamical mobile model of manipulator with multi-elbow and basement which is shown in Fig.1(b) depicting 10-links redundant manipulator whose intermediate links contact to the floor at two points, while the hand is required to draw a circle in 3D task space, comprising manipulator’s dynamics and geometrical constraint conditions, realized through the synthesization of multi-constraint condition of elbows and mobile manipulator’s motion equation. Even though a controller used for end-effector’s trajectory tracking task is simple PD controller, it has been evaluated how the contacting strategy and improve tracking accuracy and energy-saving performances.

II. MODELLING OF HYPER-REDOUNDANT MOBILE MANIPULATOR WITH CONSTRAINT
A. Manipulator’s Model with Hand’s Constraint

To make the explanation of constraint motion with multi-elbow be easily understandable, we discuss firstly about the model of the manipulator whose end-effector is contacting rigid environment without elasticity. Equation of motion of manipulator composing rigid structure of s links, and
also contact relation between manipulator’s end-effector and
definition of constraint surface should be introduced firstly.
\( L \) represents lagrangian, \( q \in \mathbb{R}^s \) represents the general coordinate, \( \tau \in \mathbb{R}^s \) represents the general input. \( u \) is the unknown constant of lagrange, \( f_t \) is the friction. Manipulator hand’s lagrange condition equation can be expressed as follows

\[
\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}} \right) - \frac{\partial L}{\partial q} = \tau + \left( \frac{\partial C}{\partial q_T} \right)^T u - \left( \frac{\partial C}{\partial r} \right)^T \frac{\dot{r}}{||r||} f_t \tag{1}
\]

Here according to the kinematic relation, manipulator hand’s position/posture vector \( r \in \mathbb{R}^s \) and scalar function, a single constraint condition \( C \) which used to express the hypersurface can be expressed as

\[
r = r(q) \tag{2}
\]

\[
C(r(q)) = 0 \tag{3}
\]

The freedom of the end-effector to move freely in the direction of non-constraint is left to be more than one, so here \( s > 1 \). If we set \( f_n \) to indicate the constraint force of manipulator hand, then the relation of \( u \) and \( f_n \) can be expressed as

\[
u = f_n/\left\| \frac{\partial C}{\partial r_T} \right\| \tag{4}
\]

\( \left\| \frac{\partial C}{\partial r_T} \right\| \) shows Euclidean norm of vector \( \frac{\partial C}{\partial r_T} \). Then manipulator’s motion equation can be derived from the combination of \( Eq(1) \) and \( Eq(4) \) with viscous friction of joints [15].

\[
M(q)\ddot{q} + h(q, \dot{q}) + g(q) + D\dot{q} = \tau + \left\{ \left( \frac{\partial C}{\partial q_T} \right)^T \left( \frac{\partial C}{\partial r_T} \right) \right\} f_n - \left( \frac{\partial r}{\partial q} \right)^T \frac{\dot{r}}{||r||} f_t \tag{5}
\]

\( M \) is inertia matrix of \( s \times s \), \( h \) and \( g \) are \( s \times 1 \) vectors which indicate the effects from coriolis force, centrifugal force and gravity, \( D \) is a \( s \times s \) matrix which indicates the coefficient of joints’ viscous friction, expressed as \( D = diag[D_1, D_2, \ldots, D_s] \), \( q \) is the joint angle and \( \tau \) is the input torque.

**B. Model with Multiple Constraints**

Here we consider a motion of a manipulator having \( s \) links whose elbows are contacting at \( p \) points to the environments defined as

\[
C_i(r_i(q)) = 0, \quad (i = 1, 2, \ldots, p) \tag{6}
\]

where \( r_i \) is the equation of position and posture of link \( i \) contacting with constraint, like \( Eq(2) \).

\[
r_i = r_i(q) \tag{7}
\]

The \( Eq(5) \) describes a motion of the manipulator whose hand is constraint. Under the situation with the \( i \)-th link contacting, then we can define following two vectors concerning \( i \)-th constraint condition \( C_i \) as follows,

\[
\left( \frac{\partial C_i}{\partial q_T} \right)^T \parallel \frac{\partial C_i}{\partial r_T} \parallel = J_{c_i}^T \tag{8}
\]

\[
\left( \frac{\partial r_i}{\partial q_T} \right)^T \parallel r_i \parallel = J_{t_i}^T \tag{9}
\]

Accumulating all the above vectors \((i = 1, 2, \ldots, p) \) when \( p \) links are contacting, the next is redefined.

\[
J_{c_i}^T = [j_{c_1}^T, j_{c_2}^T, \ldots, j_{c_p}^T] \tag{10}
\]

\[
J_{t_i}^T = [j_{t_1}^T, j_{t_2}^T, \ldots, j_{t_p}^T] \tag{11}
\]

\[
f_n = [f_{n_1}, f_{n_2}, \ldots, f_{np}]^T \tag{12}
\]

\[
f_t = [f_{t_1}, f_{t_2}, \ldots, f_{tp}]^T \tag{13}
\]

\( J_{c_i}^T, J_{t_i}^T \) are \( s \times p \) matrix, \( f_n, f_t \) are \( p \times 1 \) vectors. Considering about \( p \) constraints of the intermediate links, the manipulator’s equation of motion can be expressed as

\[
M(q)\ddot{q} + h(q, \dot{q}) + g(q) + D\dot{q} = \tau + \sum_{i=1}^{p} (J_{c_i}^T f_{ni}) - \sum_{i=1}^{p} (J_{t_i}^T f_{ti}) \tag{14}
\]

Moreover, Eq \( (6) \) is differentiated by time \( t \) two times, then we can derive the constraint condition of \( \ddot{q} \).

\[
\left[ \left( \frac{\partial C}{\partial q_T} \right)^T \right] \ddot{q} + \left( \frac{\partial C}{\partial q_T} \right) \dot{q} = 0 \tag{15}
\]

To make sure that manipulator hand is contacting with the constraint surface all the time, value of \( q(t) \) in \( Eq(14) \) has always to satisfy \( Eq(6) \) which has no relation with time \( t \), if value of \( \ddot{q} \) in \( Eq(15) \) should have the same value with \( \ddot{q} \) in \( Eq(14) \), then value of \( q(t) \) in \( Eq(14) \) and \( Eq(6) \) always keep the same regardless of time.

**C. Robot’s Dynamics Including Motors**

In this research, we want to evaluate the effects to increase the trajectory tracking accuracy and reduce the energy consumption used for countering the gravity force and other effects by bracing the intermediate links. Even though there is no robot’s motion –robot is stopping– the energy is kept to be consuming since motors of joints have to generate torques to maintain the required robot’s configuration against gravity influences. When the robot is in motion, other effects of dynamics will be added more to the gravity effect. To evaluate this kind of wasted energy consumption, we included the effects of electronic circuit of servo motor into the equation of motion of the manipulator to represent explicitly that the robot consumes energy even while stopping.
Here $v_i$ represents motor’s voltage, $R_i$ does resistance, $L_i, i_i$ do the inductance and electric current, $\tau_i$ does the angular phase of motor, $\tau_{gi}$ does the motor output torque, $\tau_{Li}$ does the load torque, $v_{gi}$ does electromotive force, $I_{mi}$ does the inertia moment of motor, $K_{Ei}$ does the constant of electromotive force, $K_{Ti}$ does the constant of torque, $d_{mi}$ does the viscous friction’s coefficient of speed reducer. The relation of those variables are shown hereunder.

\[
v_i(t) = L_i i_i(t) + R_i i_i(t) + v_{gi}(t) \\
v_{gi}(t) = K_{Ei} \dot{\theta}_i(t) \\
I_{mi} \ddot{\theta}_i = \tau_{gi}(t) - \tau_{Li}(t) - d_{mi} \dot{\theta}_i \\
\tau_{gi}(t) = K_{Ti} i_i(t)
\]

(16) (17) (18) (19)

From the relation of magnetic field and the coefficients above, $K_{Ti} = K_{Ei}(=K)$ holds for motors used. Combine Eq (17) and Eq (16), and also Eq (19) and Eq (18), we derive

\[
v_i = L_i \dot{i}_i + R_i i_i + K_{ki} \dot{\theta}_i \\
I_{mi} \ddot{\theta}_i = K_{ki} i_i - \tau_{Li} - d_{mi} \dot{\theta}_i
\]

(20) (21)

In the situation with motor and gear whose reduction ratio is $k_i$ are installed onto manipulator,

\[
\theta_i = k_i \theta_i \\
\tau_{Li} = \frac{\tau_{Li}}{k_i}
\]

(22) (23)

Combining Eq (20) and Eq (21) into equation with $i_i$ and $\tau_i$, following equations are obtained

\[
L_i \dot{i}_i = v_i - R_i i_i - K_{ki} \dot{\theta}_i \\
\tau_i = -I_{mi} k_i^2 \dot{\theta}_i + K_{ki} i_i - d_{mi} k_i^2 \dot{\theta}_i
\]

(24) (25)

Then using vector and matrix to indicate Eq (24) and (25),

\[
\begin{align*}
L \dot{q} &= v - Ri - K_m \dot{q} \\
\tau &= -J_m \dot{q} + K_m i - D_m \dot{q}
\end{align*}
\]

(26) (27)

and the definitions are shown as follow, which always have positive value.

\[
L = \text{diag}[L_1, L_2, \ldots, L_s] \\
R = \text{diag}[R_1, R_2, \ldots, R_s] \\
K_m = \text{diag}[K_{m1}, K_{m2}, \ldots, K_{ms}] \\
J_m = \text{diag}[J_{m1}, J_{m2}, \ldots, J_{ms}] \\
D_m = \text{diag}[D_{m1}, D_{m2}, \ldots, D_{ms}] \\
K_{mi} = K_{ki}, J_{mi} = I_{mi} k_i^2, D_{mi} = d_{mi} k_i^2
\]

Now substitute Eq (27) into Eq (14), we get

\[
(M(q) + J_m) \ddot{q} + h(q, \dot{q}) + g(q) + (D + D_m) \dot{q} = K_m i + J_e T f_n - J_e T f_t
\]

(28)

Similar to the same relation between Eq (14) and Eq (15), the value of $\ddot{q}$ in Eq (28) have to be identical to the value of $\ddot{q}$ in Eq (15) representing constrain condition.

### D. Robot/Motor Equation with Contact Constraint

To make sure that $\ddot{q}$ in Eq (28) and (15) be identical, constraint force $f_{n\mu}$ should be subordinately decided by simultaneous equation. Then Eq (28),(15) should be transformed as follow

\[
(M + J_m) \ddot{q} - J_e T f_n = K_m i - h - g - (D + D_m) \dot{q} - J_k T f_t
\]

(29)

\[
\left( \frac{\partial C_i}{\partial q^T} \right) \ddot{q} = \left[ \frac{\partial C_i}{\partial q} \frac{\partial C_i}{\partial q^T} \right] \dot{q}
\]

(30)

Then Eqn (29),(30),(24) can be expressed as follow. Here we assumed that friction force $f_{ini}$ is dynamic friction and define it as $f_i = 0.1 f_n(i = 1, 2, \ldots, p)$.

\[
\begin{bmatrix}
M + J_m & -J_e^T f_n & 0 & \cdots & 0 \\
\frac{\partial C_i}{\partial q} & \cdots & \frac{\partial C_p}{\partial q} & 0 & \cdots & 0 \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
0 & \cdots & 0 & L_s & \cdots & 0
\end{bmatrix}
\begin{bmatrix}
\ddot{q} \\
\frac{\partial C_1}{\partial q^T} \dot{q} \\
\vdots \\
\frac{\partial C_p}{\partial q^T} \dot{q}
\end{bmatrix}
= \begin{bmatrix}
0 \\
0 \\
\ddots \\
0
\end{bmatrix}
\begin{bmatrix}
f_{n1} \\
\vdots \\
f_{np} \\
i_1 \\
\vdots \\
i_s
\end{bmatrix}
\]

(31)

The inertia term $(M + J_m)$ is $s \times s$ matrix, the coefficient vector of constraint force $f_{n\mu}$ is $s \times 1$ vertical vector, $\partial C_i/\partial q^T$ is $1 \times s$ horizontal vector, inductance term $L$ is $s \times s$ diagonal matrix, therefore, the matrix of the first term in left side in Eq (31) is matrix of $(2s + p) \times (2s + p)$.

Then Eq (31) can be rewritten concisely using the definitions of Eq (10),(12),(26) as follow.

\[
\begin{bmatrix}
M + J_m & -J_e^T f_n \\
\frac{\partial C_i}{\partial q} & 0 & \cdots & 0 \\
0 & 0 & \cdots & 0 \\
0 & L
\end{bmatrix}
\begin{bmatrix}
\ddot{q} \\
\frac{\partial C_1}{\partial q^T} \dot{q} \\
\vdots \\
\frac{\partial C_p}{\partial q^T} \dot{q}
\end{bmatrix}
= \begin{bmatrix}
0 \\
0 \\
\ddots \\
0
\end{bmatrix}
\begin{bmatrix}
f_{n1} \\
\vdots \\
f_{np} \\
i_1 \\
\vdots \\
i_s
\end{bmatrix}
\]

(32)

where, $C$ is a vector of $C = [C_1, C_2, \ldots, C_p]^T$. Furthermore by redefining as
with respect to

Then Eq (32) can be expressed as

\[ W = M_0 \]

Let \( \omega \) be the gravity center of link 0 with respect to \( 0 \) be the gravity center of link \( i \) can be obtained by

\[ W f_0 = m_0 W P_{G0} \]

\[ W n_0 = I_0 W \omega_0 + (W I_0 W \omega_0) \]

\[ \tau_r + \tau_L = (W f_0^T W x_0) W R_0 W q_0 \]

\[ \tau_R = L_q + C_L \dot{q}_L + C_R \dot{q}_R \]

\[ \tilde{\tau}_R \text{ and } \tilde{\tau}_L , \text{ which are calculated from } W f_0 \text{ and } W n_0 , \text{ contain } V_0, \omega_0, \text{ and } f_0 \text{ as variables in them.} \]

III. FORWARD DYNAMICS CALCULATION

To calculate \( M^*, b \) in Eq (35), we need to first calculate \( M, h, g \). Here we can notice that \( M, h, g \) are included Eq (28) that describes the dynamics of non-constraint, and those can be calculated numerically and recursively through forward dynamics calculation [16] by exploiting the inverse dynamics calculation called "Newton Euler" Method [17]. Because \( M \) is \( 10 \times 10 \) matrix when the hyper-redundant manipulator including \( 10 \) links, resulting in a large amount of computation to calculate each element of \( M \) by using lagrange method. This implies that analytical deriving Eq
Then considering Eq (51) and Eq (52), we substitute $\ddot{\bar{r}} \cdot b$ so that we can calculate $K$ and $D$ as the function of $\bar{M}$ and $\bar{b}$.

First of all, Eq (28) should be set as hereunder.

$$ M_J \ddot{q} + b_J = \ddot{\bar{r}} $$

(51)

Here

$$ M_J = M(q) + J_m $$

$$ b_J = h(q, \dot{q}) + \dot{(D + D_m)} \dot{q} $$

$$ \ddot{\bar{r}} = K_m \dot{q} + \dot{J}_cT f_n - J_t^T \dot{f}_t $$

With forward motion analysis, Eq (51) should be calculated by Newton-Euler method from the bottom link to upper link until the manipulator’s hand, and also with the motion analysis of backward calculation, we get equation of motion of $i$-th link Eq (52).

$$ \ddot{\bar{r}}_i = \ddot{\bar{z}}^T \dot{n}_i + J_{mi} \ddot{q}_i + (D_i + D_{mi}) \dot{q}_i $$

(52)

Therefore, the motion Eq (51) can be used to inverse dynamics calculation $\ddot{\bar{r}} = [\ddot{r}_1, \ddot{r}_2, \cdots, \ddot{r}_n]^T$ in Eq (52). This inverse calculation can be described as $\ddot{\bar{r}} = p(q, \dot{q}, \ddot{q}, g)$.

Then considering Eq (51) and Eq (52),

$$ M_J \ddot{q} + b_J = p(q, \dot{q}, \ddot{q}, g) $$

(53)

Substitute $\ddot{q} = 0$ into Eq (53):

$$ b_J = p(q, \dot{q}, 0, g) $$

(54)

so $b_J$ can be calculated. Next substitute $g = 0, \dot{q} = 0, \ddot{q} = e_i (i = 1, 2, \cdots, s)$ into Eq (53), then the $b_J = 0$ :

$$ m_i = M_J e_i = p(q, 0, e_i, 0) $$

(55)

here we can calculate $m_i$ defined as the component vector of the $i$-th column in inertia matrix $M$, $e_i$ is a $l \times 1$ matrix in which the $i$-th element is 1 and others are all 0 like $e_i = [0, 0, \cdots, 1, 0, \cdots, 0]^T$. So with Eq (55) $M_J = [m_1, m_2, \cdots, m_l]$ can be calculated one by one separately.

Thus up to now, we have calculated the $M_J$ and $b_J$. Back to the Eq (33), the $M^*$ can be calculated while the constraint condition is given. Moreover, the inverse of $M^*$ can be also calculated due to invertible for $M^*$.

IV. TRAJECTORY TRACKING SIMULATION

In this section we will introduce the trajectory tracking simulation results. The input voltage of PD controller has been set as follows.

$$ v = K_p (q_d - q) + K_d (\dot{q}_d - \dot{q}) $$

(56)

$K_p$ and $K_d$ are both $s \times s$ diagonal matrix which indicates a position gain and a velocity gain, $q_d, \dot{q}_d$ are the desired joint angle and joint angular velocity, respectively.

Simulation’s condition has been set as: each link’s mass is $m_i = 0.1[kg]$, length is $l_i = 0[m]$, $l_j = 0.3[m]$, radius of cylindrical link is $r_i = 0.01[m]$, proportional gain is $k_p = 500$, velocity gain is $k_v = 20$, viscous friction coefficient of joint is $D_i = 0.5$, torque constant is $K_i = 0.203$, resistance is $R_i = 1.1[\Omega]$, inductance is $L_i = 0.0017[H]$, inertia moment of motor is $I_{mi} = 0.000164$, reduction ratio is $k_i = 3.0$, viscous friction coefficient of reducer is $d_{mi} = 0.01$ and these parameters are given by actual motor’s specifications. Initial condition of each link: $q_1(0) = 0, q_2(0) = 0.25\pi, q_3(0) = 0.5\pi, q_4(0) = -0.5\pi, q_5(0) = 0.25\pi, q_6(0) = 0.25\pi, q_7(0) = -0.5\pi, q_8(0) = 0.25\pi, q_9(0) = 0\pi$. Moreover, the desired trajectory’s parameters have been set as: $q_{d2}(t) = 0.25\pi, q_{d3}(t) = 0.5\pi, q_{d4}(t) = -0.5\pi, q_{d5}(t) = 0.25\pi, q_{d6}(t) = 0.25\pi, q_{d7}(t) = -0.5\pi, q_{d8}(t) = 0.25\pi$, and the trajectory has been set as a circle with radius is $0.1[m]$, center is $(x, y, z) = (1.5, 1.5, 0.4)$, target which is tracked by the manipulator hand will rotate in counterclockwise along this circle trajectory. The constraint condition is set as $C = z = 0$. Simulation has been done under three situations and the simulation model is also shown as (Fig.3).

Fig. 3. The simulation model of mobile manipulator

1) trajectory tracking motion with two elbows contacting at joint 4 and joint 7;
2) trajectory tracking motion with just one elbow contacting at joint 7;
3) trajectory tracking motion with just one elbow contacting at joint 4;
4) trajectory tracking motion with no elbow contacting at all.

Simulation results are shown in Fig.4~Fig.8. Fig.4 shows that manipulator hand’s trajectory in $xy$ coordinate under three different simulation condition, in the same way, Fig.6 shows that manipulator hand’s trajectory in $yz$ coordinate, and Fig.5 shows that manipulator hand’s trajectory in $xz$ coordinate, the simulation time is from 0 to 10[s]. Fig.7 shows that all manipulator links’ total amount of cost electric energy during the whole simulation. From Fig.5, we can tell that the manipulator hand can track the circle trajectory more accurately with more restraint elbows. Especially, in $z$ axis, along with gravity’s direction, motion of manipulator hand without restraint elbow is affected by the nutation of each link, the trajectory tracking has not been perfectly accomplished. Moreover, from Fig.7 and Fig.8, even in case of doing same work by manipulator, the cost electric energy is bigger in the motion done by the manipulator without restraint elbow.
condition. We think the usage of contacting motion with environment is promising as a new robot motion control strategy.

REFERENCES