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Shape-grinding by Direct Position / Force Control with On-line Constraint Estimation

Mamoru Minami and Weiwei Xu

***Abstract*— Based on the analysis of the interaction between a manipulator's hand and a working object, a model representing the constrained dynamics of the robot is first discussed. The constrained forces are expressed by an algebraic function of states, input generalized forces, and constraint condition, and then direct position / force controller without force sensor is proposed based on the algebraic relation. To give the grinding system the ability to adapt to any object shape being changed by the grinding, we added estimating function of the constraint condition in real time for the adaptive position / force control. Evaluations through simulations by fitting the changing constraint surface with spline functions, indicate that reliable position / force control and shape-grinding can be achieved by the proposed controller.**

I. Introduction

Many researches have discussed on the force control of robots for contacting tasks. Most force control strategies are to use force sensors [1], where the reliability and accuracy are limited since the work-sites of the robot are filled with noise and thermal disturbances. Force sensors could lead to the falling of the structure stiffness of manipulators, which is one of the most essential defects for manipulators executing grinding tasks. To solve these problems, some approaches without any force sensor have been presented [2]. To ensure the stabilities of the constrained motion, force and position control have utilized Lyapunov's stability analysis under the inverse dynamic compensation. Their force control strategies have been explained intelligibly in books [3]-[5].

However, insofar as we survey the controllers introduced in the books or published papers are not based on the algebraic function of states and input generalized forces derived from the relation between the constraint condition and the equation of dynamics. So we discuss first a strategy for simultaneous control of the position and force without any force sensors, where the equation of dynamics in reference to the constrained force has been reformulated. The constrained force is derived from the equation of dynamics and the constrained equation is defined as an explicit algebraic function of states and input generalized forces, which means that force information can be obtained by calculation rather than by force sensing. Eq. (1), which has been pointed out by Hemami in the analysis of biped walking robot, denotes also the kinematical algebraic relation of the controller, when robot's end-effecter being in touch with a surface in 3-D space:

$$F_n = a(\boldsymbol{x}_1, \boldsymbol{x}_2) - \boldsymbol{A}(\boldsymbol{x}_1)\boldsymbol{\tau}, \quad (1)$$

where, F_n is the exerting force on the constrained surface. $\boldsymbol{x}_1$ and $\boldsymbol{x}_2$ are state variables. $a(\boldsymbol{x}_1, \boldsymbol{x}_2)$ and $\boldsymbol{A}(\boldsymbol{x}_1)$ are scalar function and vector one defined in following section. $\boldsymbol{\tau}$ is input torque. The above algebraic equation has been known in robotic field. It had been applied the first time to the sensing function of exerting force by Peng. As a new control law, the controller doesn't include any force feedback sensors but realizes simultaneous control of position and force in the constrained motions and is different from the traditional ones [1].

A strategy to control force and position proposed in this paper is also based on Eq. (1). Contrarily to Peng's Method to use Eq. (1) as a force sensor, we used the equation for calculating $\boldsymbol{\tau}$ to achieve a desired exerting force F_{nd}. Actually, the strategy is based on two facts of Eq. (1) that have been ignored for a long time. The first fact is that the force transmission process is an immediately process being stated clearly by Eq. (1) providing that the manipulator's structure is rigid. Contrarily, the occurrence of velocity and position is a time-consuming process. By using this algebraic relation, it's possible to control the exerting force to the desired one without time lag. Another important fact is the input generalized forces have some redundancy against the constrained generalized forces in the constrained motion. Based on the above analysis, we had confirmed our force / position control method can realize the grinding task through real grinding robot.

The problem to be solved in our approach is that the mathematical expression of algebraic constraint condition should be defined in the controller instead of the merit of not using force sensor. Grinding task requires on-line estimation of changing constraint condition since the grinding is the action to change the constraint condition. In this presentation, we estimate the object's surface using the grinder as a touch sensor. In order to give the system the ability to grind any working object into any shape, we focus on how to update the constraint condition in real time, obtaining the result that spline function is best for on-line shape estimation. Based on the above preparation we constructed a simulator to evaluate the proposed shape-grinding system, resulting in having proven the validity of our system to have the performance to adapt for grinding to desired-shape without force sensor.

Mamoru Minami is with Faculty of Engineering, University of Fukui, Fukui, Japan minami@rc.his.fukui-u.ac.jp

Weiwei Xu is with Graduate school of Engineering, University of Fukui, Fukui, Japan xuweiwei@rc.his.fukui-u.ac.jp

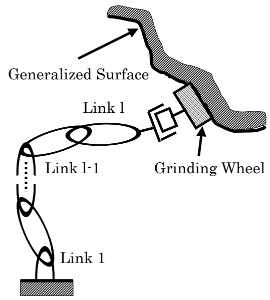

Fig. 1. A Grinding Robot

II. Analysis of Grinding Task

Generally speaking, the grinding power is related to the metal removal rate(weight of metal being removed within unit time), which is determined by the depth of cut, the width of cut, the linear velocity of the grinding wheel, the feed rate and so on. There are many empirical formulae available for the determination of grinding power, and the desired force trajectory can then be planned according to the power. The normal grinding force F_n is exerted in the perpendicular direction of the surface. It is a significant factor that affects ground accuracy and surface roughness of workpiece. The value of it is also related to the grinding power or directly to the tangential grinding force as

$$F_t = \mathrm{K_t} F_n, \tag{2}$$

where, $\mathrm{K_t}$ is an empirical coefficient, F_t is the tangential grinding force.

The axial grinding force F_s is proportional with the feed rate, and is much smaller than the former force.

Eq. (2) is based on the situation that position of the grinding cutter is controlled like currently used machining center. But when a robot is used for the grinding task, the exerting force to the object and the position of the grinding cutter should be controlled simultaneously. The F_n is generally determined by the constrained situation, and it is not suitable to apply Eq. (2) to grinding motion by the robots.

III. Modeling

A. Constrained Dynamic Systems

Hemami and Wyman have addressed the issue of control of a moving robot according to constraint condition and examined the problem of the control of the biped locomotion constrained in the frontal plane. Their purpose was to control the position coordinates of the biped locomotion rather than generalized forces of constrained dynamic equation involved the item of generalized forces of constraints. And the constrained force is used as a determining condition to change the dynamic model from constrained motion to free motion of the legs. In this paper, the grinding manipulator shown in Fig. 1, whose end-point is in contact with the constrained surface, is modelled according Eq. (3) with Lagrangian equations of motion in term of the constraint forces, refering to what Hemami and Arimoto have done:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\boldsymbol{q}}}\right)-\left(\frac{\partial L}{\partial \boldsymbol{q}}\right)=\boldsymbol{\tau}+\boldsymbol{J_c}^T(\boldsymbol{q})F_n-\boldsymbol{J_r}^T(\boldsymbol{q})F_t, \tag{3}$$

where, $\boldsymbol{J_c}$ and $\boldsymbol{J_r}$ satisfy,

$$\boldsymbol{J_c}=\frac{\partial C}{\partial \boldsymbol{q}}/\parallel\frac{\partial C}{\partial \boldsymbol{r}}\parallel=\frac{\partial C}{\partial \boldsymbol{r}}\,\tilde{\boldsymbol{J}}_r/\parallel\frac{\partial C}{\partial \boldsymbol{r}}\parallel,$$

$$\tilde{\boldsymbol{J}}_r=\frac{\partial \boldsymbol{r}}{\partial \boldsymbol{q}},\qquad \boldsymbol{J}_r^T=\tilde{\boldsymbol{J}}_r^{\,T}\dot{\boldsymbol{r}}/\parallel\dot{\boldsymbol{r}}\parallel,$$

$\boldsymbol{r}$ is the l position vector of the hand and can be expressed as a kinematic equation ,

$$\boldsymbol{r}=\boldsymbol{r}(\boldsymbol{q}). \tag{4}$$

L is the Lagrangian function, $\boldsymbol{q}$ is $l(\geq 2)$ generalized coordinates, $\boldsymbol{\tau}$ is l inputs. The discussing robot system does not have kinematical redundancy. C is a scalar function of the constraint, and is expressed as an equation of constraints

$$C(\boldsymbol{r}(\boldsymbol{q}))=0, \tag{5}$$

F_n is the constrained force associated with C and F_t is the tangential disturbance force.

Eq. (3) can be derived to be

$$\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}}+\boldsymbol{H}(\boldsymbol{q},\dot{\boldsymbol{q}})+\boldsymbol{G}(\boldsymbol{q})$$
$$=\boldsymbol{\tau}+\boldsymbol{J}_c^T(\boldsymbol{q})F_n-\boldsymbol{J}_r^T(\boldsymbol{q})F_t, \tag{6}$$

where $\boldsymbol{M}$ is an $l\times l$ matrix, $\boldsymbol{H}$ and $\boldsymbol{G}$ are l vectors. The state variable $\boldsymbol{x}$ is constructed by adjoining $\boldsymbol{q}$ and $\dot{\boldsymbol{q}}$: $\boldsymbol{x}=(\boldsymbol{x}_1^T,\boldsymbol{x}_2^T)^T$=$(\boldsymbol{q}^T,\dot{\boldsymbol{q}}^T)^T$. The state-space equation of the system are

$$\begin{aligned}\dot{\boldsymbol{x}}_1 &= \boldsymbol{x}_2,\\ \dot{\boldsymbol{x}}_2 &= -\boldsymbol{M}^{-1}(\boldsymbol{H}(\boldsymbol{x}_1,\boldsymbol{x}_2)+\boldsymbol{G}(\boldsymbol{x}_1))\\ &\quad +\boldsymbol{M}^{-1}(\boldsymbol{\tau}+\boldsymbol{J}_c^T(\boldsymbol{x}_1)F_n-\boldsymbol{J}_r^T\boldsymbol{x}_1)F_t),\end{aligned} \tag{7}$$

or in the compact form

$$\dot{\boldsymbol{x}}=\boldsymbol{F}(\boldsymbol{x},\boldsymbol{\tau},F_n,F_t), \tag{8}$$

where the dimension of $\boldsymbol{x}$ is $n=2l$. In order to control the system(Eq.(8)) with constraints condition(Eq. (5)) , it can be done firstly by differentiating the constraint Eq. (5) twice with respect to time and rewriting the result in terms of $\boldsymbol{x}$:

$$\boldsymbol{D}(\boldsymbol{x})\dot{\boldsymbol{x}}=0, \tag{9}$$

where, $\boldsymbol{D}(\boldsymbol{x})$ is a n vector considering that the constrained motion of the system is orthogonal. Premultiplying Eq. (8) by $\boldsymbol{D}(\boldsymbol{x})$ derived from Eq. (9),

$$\boldsymbol{D}(\boldsymbol{x})\boldsymbol{F}(\boldsymbol{x},\boldsymbol{\tau},F_n,F_t)=0. \tag{10}$$

This is a linear equation about the unknown constrained force F_n , combining the constrained equation and the equation of motion.Eq. (10) can be uniquely solved for F_n as a function of the state $\boldsymbol{x}$ and input $\boldsymbol{\tau}$,

$$-\left[\frac{\partial}{\partial \boldsymbol{q}}\left(\frac{\partial C}{\partial \boldsymbol{q}}\right)\dot{\boldsymbol{q}}\right]\dot{\boldsymbol{q}}+\left(\frac{\partial C}{\partial \boldsymbol{q}}\right)\boldsymbol{M}^{-1}(\boldsymbol{H}(\boldsymbol{q},\dot{\boldsymbol{q}})+\boldsymbol{G}(\boldsymbol{q})+\boldsymbol{J_r}^T F_t)$$
$$-\left(\frac{\partial C}{\partial \boldsymbol{q}}\right)\boldsymbol{M}^{-1}\boldsymbol{\tau}=\left[\left(\frac{\partial C}{\partial \boldsymbol{q}}\right)\boldsymbol{M}^{-1}\left(\frac{\partial C}{\partial \boldsymbol{q}}\right)^T\right]F_n/\parallel\frac{\partial C}{\partial \boldsymbol{r}}\parallel, \tag{11}$$

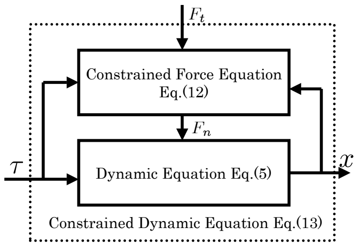

Fig. 2. Model of Constrained Dynamic System

because the value of $(\frac{\partial C}{\partial \boldsymbol{q}})\boldsymbol{M}^{-1}(\frac{\partial C}{\partial \boldsymbol{q}})^T(=m_c)$ is always positive, hence it is also invertible. In this case, from Eq. (11) F_n can be expressed as

$$F_n = F_n(\boldsymbol{x}, \boldsymbol{\tau}, F_t), \tag{12}$$

or in a more detailed form

$$F_n = [(\frac{\partial C}{\partial \boldsymbol{q}})\boldsymbol{M}^{-1}(\frac{\partial C}{\partial \boldsymbol{q}})^T]^{-1} \parallel \frac{\partial C}{\partial \boldsymbol{r}} \parallel$$

$$\{-[\frac{\partial}{\partial \boldsymbol{q}}(\frac{\partial C}{\partial \boldsymbol{q}})\dot{\boldsymbol{q}}]\dot{\boldsymbol{q}}+(\frac{\partial C}{\partial \boldsymbol{q}})\boldsymbol{M}^{-1}(\boldsymbol{H}(\boldsymbol{q},\dot{\boldsymbol{q}})+\boldsymbol{G}(\boldsymbol{q})+\boldsymbol{J_r}^T F_t)\}$$

$$-[(\frac{\partial C}{\partial \boldsymbol{q}})\boldsymbol{M}^{-1}(\frac{\partial C}{\partial \boldsymbol{q}})^T]^{-1} \parallel \frac{\partial C}{\partial \boldsymbol{r}} \parallel \{(\frac{\partial C}{\partial \boldsymbol{q}})\boldsymbol{M}^{-1}\}\boldsymbol{\tau}$$

$$\triangleq a(\boldsymbol{x}_1, \boldsymbol{x}_2) + \boldsymbol{A}(\boldsymbol{x}_1)\boldsymbol{J}_r^T F_t - \boldsymbol{A}(\boldsymbol{x}_1)\boldsymbol{\tau}, \tag{13}$$

where, $a(\boldsymbol{x}_1, \boldsymbol{x}_2)$ is a scalar representing the first term in the expression of F_n, and $\boldsymbol{A}(\boldsymbol{x}_1)$ is an l vector to represent the coefficient vector of $\boldsymbol{\tau}$ in the same expression. Eq. (8) and Eq. (12) compose a constrained system that can be controlled, if $F_n = 0$, describing the unconstrained motion of the system.

Substituting Eq. (13) into Eq. (7), the state equation of the system including the constrained force (as $F_n > 0$) can be rewritten as

$$\dot{\boldsymbol{x}}_1 = \boldsymbol{x}_2,$$

$$\dot{\boldsymbol{x}}_2 = -\boldsymbol{M}^{-1}[\boldsymbol{H}(\boldsymbol{x}_1, \boldsymbol{x}_2) + \boldsymbol{G}(\boldsymbol{x}_1) - \boldsymbol{J}_c^T(\boldsymbol{x}_1)a(\boldsymbol{x}_1, \boldsymbol{x}_2)]$$

$$+\boldsymbol{M}^{-1}[(\boldsymbol{I} - \boldsymbol{J}_c^T\boldsymbol{A})\boldsymbol{\tau} + (\boldsymbol{J}_c^T\boldsymbol{A} - \boldsymbol{I})\boldsymbol{J}_r^T F_t], \tag{14}$$

which is denoted as a model of the constrained dynamic system in Fig. 2. Solutions of these dynamic equation always satisfy the constrained condition Eq. (5).

B. Shape grinding

In the past, we did the experiment when working surface was flat, so we can just do flat grinding. Now we want to grind the work-piece into the one with different kinds of shapes, for example, grinding the flat surface into a curved one, just like Fig. 4. In Fig. 4, we can find that the desired working surface is prescribed (it can be decided by us.), which means the desired constrained condition C_d is known, so

$$C_d = y - f_d(x) = 0 \tag{15}$$

But the constrained condition $C^{(j)}$ $(j = 1, 2, \cdots, d-1)$ changed by the previous grinding which is in the Dynamic System of Fig. 3 is hard to defined as an initial condition. So we define

$$C^{(j)} = y - f^{(j)}(x) = 0 \tag{16}$$

where, y is the y position of manipulator's end-effector in the coordinates Σw depicted in Fig. 4 and we assume $C^{(1)}$ is known, that is to say, $f^{(1)}(x)$ is initially defined. $f^{(j)}(x)$ is the working surface remained by i-th grinding. And $f^{(j)}(x)$ is a function passing through all points, (x_1, $f^{(j)}(x_1)$), (x_2, $f^{(j)}(x_2)$), $\cdots$, (x_p, $f^{(j)}(x_p)$), these observed points representing the (j)-th constraint condition obtained from the grinding tip position since we proposed previously the grinding tip used for the touching sensor of ground new surface. Here we assume $f^{(j)}(x)$ could be represented by a polynomial of $(p-1)$-th order of x. Given the above p points, we can easily decide the parameters of polynomial function $y = f^{(j)}(x)$. If the current constrained condition can be got successfully, which means the current working surface $f^{(j)}(x)$ can be detected correctly, the distance from the current working surface to the desired working surface which is expressed as $\Delta h^{(j)}$ shown in Fig. 4 can be obtained easily.

$$\Delta h^{(j)}(x_i) = f^{(j)}(x)\big|_{x=x_i} - f_d(x)\big|_{x=x_i} \tag{17}$$

In this case, we can obviously find that the desired constrained force should not be a constant. It should be changed while $\Delta h^{(j)}$ changes. So we redefine the desired constrained force $F_{nd}^{(j)}$ as a function of $\Delta h^{(j)}$, shown as follows:

$$F_{nd}^{(j)}(x_i) = k\Delta h^{(j)}(x_i) \tag{18}$$

where, k is a constant.

We can describe the grinding procedure as the removal length in y direction is proportional to the exerting force $F_n^{(j)}(x_i)$, which is determined through Eq. (13) and Eq. (24), then new-ground surface $f^{(j+1)}(x_i)$ can be obtained through exerted force $F_n^{(j)}(x_i)$ and previous constraint $f^{(j)}(x_i)$ as

$$f^{(j+1)}(x_i) - f^{(j)}(x_i) = k' F_n^{(j)}(x_i) \tag{19}$$

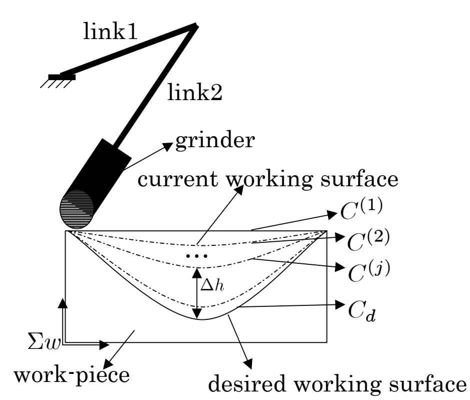

Fig. 4. The model of shape grinding

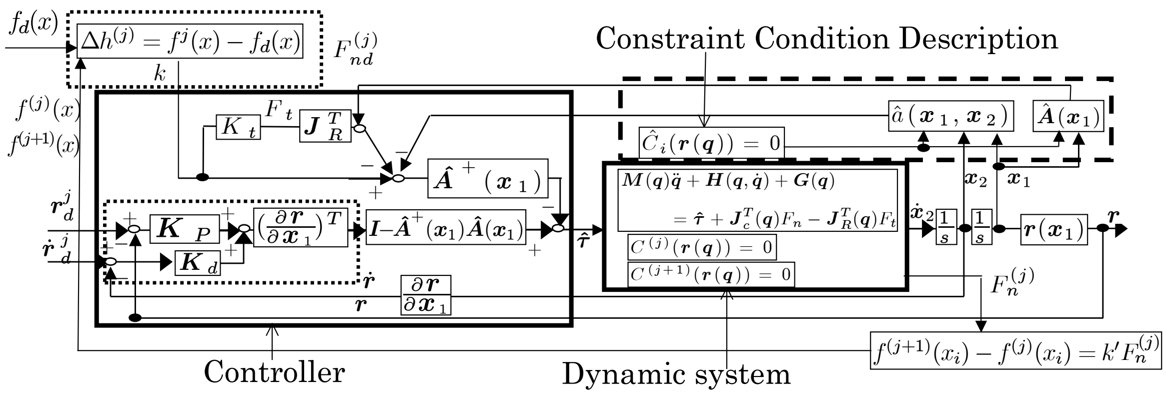

Fig. 3. Shape-grinding position / force control system

where, k' is a constant. A condition that the new object shape $f^{(j+1)}(x_i)$ have to satisfy, i.e.,

$$y = f^{(j+1)}(x_i) \tag{20}$$

Then $C^{(j+1)}$ can also be known:

$$C^{(j+1)} = y - f^{(j+1)}(x) = 0 \tag{21}$$

So, starting from $C^{(1)}$, all of $C^{(j)}$ can be decided. What we want to emphasize is C_i represents the resulted ground shape of the object defined in the shape-grinding simulator.

IV. Force and Position Controller

A. Controller using predicted constraint condition

Reviewing the dynamic equation(Eq. (3)) and constraint condition(Eq. (5)), it can be found that as $l > 1$, the number of input generalized forces is more than that of the constrained forces. From this point and Eq. (13) we can claim that there is some redundancy of constrained force between the input torque $\boldsymbol{\tau}$, and the constrained force F_n. This condition is much similar to the kinematical redundancy of redundant manipulator. Based on the above argument and assuming that, the parameters of the Eq. (13) are known and its state variables could be measured, and $a(\boldsymbol{x}_1, \boldsymbol{x}_2)$ and $\boldsymbol{A}(\boldsymbol{x}_1)$ could be calculated correctly, which means that the constraint condition $C = 0$ is prescribed. As a result, a control law is derived and can be expressed as

$$\boldsymbol{\tau} = -\boldsymbol{A}^+(\boldsymbol{x}_1)\Big\{F_{nd} - a(\boldsymbol{x}_1, \boldsymbol{x}_2) - \boldsymbol{A}(\boldsymbol{x}_1)\boldsymbol{J}_R^T F_t\Big\} + (\mathbf{I} - \boldsymbol{A}^+(\boldsymbol{x}_1)\boldsymbol{A}(\boldsymbol{x}_1))\boldsymbol{k}, \tag{22}$$

where $\mathbf{I}$ is a $l \times l$ identity matrix, F_{nd} is the desired constrained forces, $\boldsymbol{A}(\boldsymbol{x}_1)$ is defined in Eq. (13) and $\boldsymbol{A}^+(\boldsymbol{x}_1)$ is the pseudoinverse matrix of it, $a(\boldsymbol{x}_1, \boldsymbol{x}_2)$ is also defined in Eq. (13) and $\boldsymbol{k}$ is an arbitrary vector which is defined as

$$\boldsymbol{k} = \tilde{\boldsymbol{J}}_r^T(\boldsymbol{q})\Big\{\boldsymbol{K}_p(\boldsymbol{r}_d - \boldsymbol{r}) + \boldsymbol{K}_d(\dot{\boldsymbol{r}}_d - \dot{\boldsymbol{r}})\Big\}, \tag{23}$$

where $\boldsymbol{K}_p$ and $\boldsymbol{K}_d$ are gain matrices for position and the velocity control by the redundant degree of freedom of $\boldsymbol{A}(\boldsymbol{x}_1)$, $\boldsymbol{r}_d(\boldsymbol{q})$ is the desired position vector of the end-effector along the constrained surface and $\boldsymbol{r}(\boldsymbol{q})$ is the real position vector of it. Eq. (23) describes the 2-link rigid manipulator's arm compliance, we have to set $\boldsymbol{K}_p$ and $\boldsymbol{K}_d$ with a reasonable value, otherwise high-frequency response of position error will appear. The controller presented by Eq. (22) and Eq. (23) assumes that the constraint condition $C = 0$ be known precisely even though the grinding operation is a task to change the constraint condition. This looks like to be a contradiction, so we need to observe time-varying constraint conditions in real time by using grinding tip as a touch sensor.

The time-varying condition is estimated as an approximate constrained function by position of the manipulator hand, which is based on the estimated constrained surface location. The estimated condition is denoted by $\hat{C} = 0$. Hence, $a(\boldsymbol{x}_1, \boldsymbol{x}_2)$ and $\boldsymbol{A}(\boldsymbol{x}_1)$ including $\partial\hat{C}/\partial\boldsymbol{q}$ and $\partial/\partial\boldsymbol{q}(\partial\hat{C}/\partial\boldsymbol{q})$ are changed to $\hat{a}(\boldsymbol{x}_1, \boldsymbol{x}_2)$ and $\hat{\boldsymbol{A}}(\boldsymbol{x}_1)$ as shown in Eq. (25), Eq. (26). They were used in the later simulations of the unknown constrained condition. As a result, a controller based on the estimated constrained condition is given as

$$\hat{\boldsymbol{\tau}} = -\hat{\boldsymbol{A}}^+(\boldsymbol{x}_1)\Big\{F_{nd} - \hat{a}(\boldsymbol{x}_1, \boldsymbol{x}_2) - \hat{\boldsymbol{A}}(\boldsymbol{x}_1)\boldsymbol{J}_R^T F_t\Big\} + (\mathbf{I} - \hat{\boldsymbol{A}}^+(\boldsymbol{x}_1)\hat{\boldsymbol{A}}(\boldsymbol{x}_1))\boldsymbol{k}, \tag{24}$$

$$m_c{}^{-1}\Big\| \frac{\partial \hat{C}}{\partial \boldsymbol{r}} \Big\| \Big\{-\Big[\frac{\partial}{\partial \boldsymbol{q}}\Big(\frac{\partial \hat{C}}{\partial \boldsymbol{q}}\Big)\dot{\boldsymbol{q}}\Big]\dot{\boldsymbol{q}} + \Big(\frac{\partial \hat{C}}{\partial \boldsymbol{q}}\Big)\boldsymbol{M}^{-1}(\boldsymbol{h}+\boldsymbol{g})\Big\} \triangleq \hat{a}(\boldsymbol{x}_1, \boldsymbol{x}_2) \tag{25}$$

$$m_c{}^{-1}\Big\| \frac{\partial \hat{C}}{\partial \boldsymbol{r}} \Big\| \Big\{\Big(\frac{\partial \hat{C}}{\partial \boldsymbol{q}}\Big)\boldsymbol{M}^{-1}\Big\} \triangleq \hat{\boldsymbol{A}}(\boldsymbol{x}_1) \tag{26}$$

Figure 3 illustrates a control system constructed according to the above control law that consists of a position feedback control loop and a force feedfoward control. It can be found from Eq. (13) and Eq. (24) that the constrained force always equals to the desired one explicitly if the estimated constraint condition equals to the real one, i.e., $C = \hat{C}$ and $F_t = 0$. This is based on the fact that force transmission is an instant process. In the next section, we will introduce several prediction methods which are used to get $\hat{C}_i$ in current time.

The experiment when the constraint is known have been done successfully in Fig. 5. The maximum position error is about 8[mm], and the maximum force error is 0[N]. Based on the experiment when the constraint is known, we propose the method when the constraint is unknown as follows.

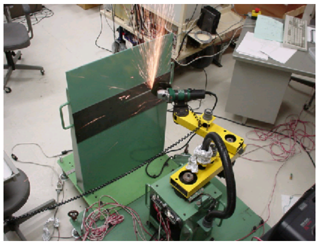

Fig. 5. The experiment when the constraint is known

B. On-line Estimation of Constraint

As it is stated in former section, we had done the grinding experiment when working surface was flat, and now curved surface shape-grinding is proposed to be solved in our research. But how to predict the unknown constraint surface is the nodus and key point. Since constraint surface is unknown, we concentrate our efforts on considering solutions and comparing the effection, finally select a best solution to accomplish this new experiment task. We proposed three ways to estimate the unknown constraint surface, it is fitted with linear function, quadratic function, and spline curve. Three simulations have been done to base on different constraint conditions. Here, an unknown constrained condition is estimated as following,

(Assumptions)

1. The end point position of the manipulator during performing the grinding task can be surely measured and updated.
2. The grinding task is defined in $x-y$ plane.
3. When beginning to work, the initial condition of the end-effector is known and it has touched the work object.
4. The chipped and changed constraint condition can be approximated by connections of minute sections.

Three methods which are fitting by linear function, quadratic function and spline function had been used to get the online estimation of the unknown constrained condition. Here we just introduce the spline curve fitting.

1) Fitting by quadratic spline curve: The unknown constrained condition, which is represented in Fig. 3, is estimated and expressed as,

$$\hat{C}_{i+1} = y - [A_i(x-x_{i-1})^2 + B_i(x-x_{i-1}) + C_i] \quad (27)$$

The end-effector position at time $(i-1)\Delta t$, $i\Delta t$ are denoted respectively as (x_{i-1}, y_{i-1}), (x_i, y_i).

The quadratic spline curve denoted as

$$S_i(x) = A_i(x-x_{i-1})^2 + B_i(x-x_{i-1}) + C_i, \quad x \in [x_{i-1}, x_i](i = 1, 2, 3 \cdots n) \quad (28)$$

The constrained condition $\hat{C}_{i+1} = y - (A_i(x-x_{i-1})^2 + B_i(x-x_{i-1}) + C_i)$ can be determined. Also, we can get the coefficients of the spline curve uniquely as follows.

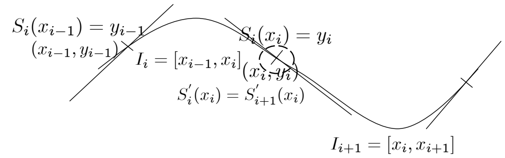

Fig. 6. Fitting by quadratic spline curve

Firstly, let $S_i(x)$ satisfy the following conditions shown in Fig. 6.

(A)Go through two ends of the interval

$$y_{i-1} = S_i(x_{i-1}) \quad (29)$$

$$y_i = S_i(x_i) \quad (30)$$

(B)First-order differential of the spline polynomials are equal at the end-point of adjoined function.

$$\left.\frac{dS_{i+1}(x)}{dx}\right|_{x=x_i} = \left.\frac{dS_i(x)}{dx}\right|_{x=x_i} S_{i+1}'(x_i) = S_i'(x_i) \quad (31)$$

Inputting (28) into (29), (30) and (31), we can obtain:

$$C_i = y_{i-1}, (i = 1, 2, \cdots, n) \quad (32)$$

$$B_{i+1} = 2u_i - B_i, (i = 1, 2, \cdots, n-1) \quad (33)$$

$$A_i = \frac{B_{i+1} - B_i}{2h_i}, (i = 1, 2, \cdots, n-1) \quad (34)$$

Where, $h_i = x_i - x_{i-1}$, $u_i = \frac{y_i - y_{i-1}}{h_i}$. From the above-mentioned result, the constrained conditional expression $\hat{\boldsymbol{C}}_{i+1}$ can be updated step by step.

In this point, we can see that the spline curve is defined by two points and a derivative at some point. Comparing to the quadratic function fitting and linear function fitting, fitting by quadratic spline curve can more precisely represent the ground surface because derivative information at hand-touching position, i.e., tangential direction of the surface is being included in the spline representation. So we can say method of "fitting by quadratic spline curve" is the best. It will be verified by simulations later.

V. SIMULATION

A planar two-link manipulator is applied for simulation so as to examine the behaviour of the proposed controller. The goals were to examine the feasibility of the proposed method with regard to the accuracy and stability. Three simulations have been done based on different constraint conditions.

The model of grinding robot manipulator used in the simulation is shown in Fig.4, whose parameters are: length of link 1 is 0.3[m], length of link 2 is 0.5[m], and the mass of link 1 is 12.28[kg], the mass of link 2 is 7.64[kg]. The end-effector velocity, 0.01[m/s], the desired constrained force, F_{nd} = 5[N], grinding resistance, F_t = 0[N].

The desired constrained surface is denoted as

$$f(x) = p - k\cos(\omega x) \quad (35)$$

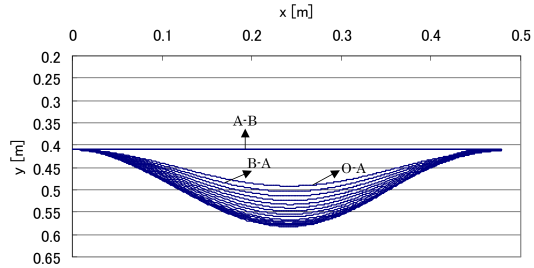

Fig. 7. The trajectory of simulation

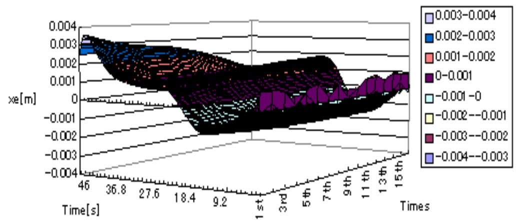

Fig. 8. Error of x position

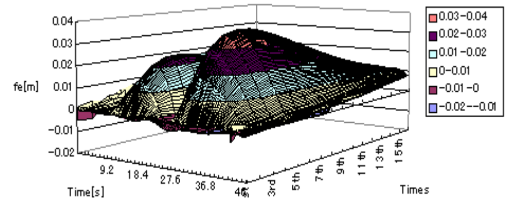

Fig. 9. Force Error

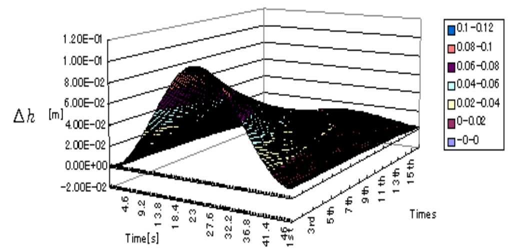

Fig. 10. The changes of Δh

A. Shape Grinding-simulation

Since we have known that the spline curve fitting is the best, we can use it to make the system adapt to the shape-changing grinding represented in Fig. 4. In this simulation, the constant k shown in Eq. (18) is 50, and $k^{'}$ shown in Eq. (19) is 0.002. According to the real condition, we changed the coefficient of desired constraint condition shown in Eq. (35), which is p=0.50, k=0.09, ω=13. The trajectory of simulation is showing Fig. 7. The trajectory named $O-A$ is the first grinding, and then go back to the starting point through a line which is named $A-B$. The second grinding trajectory is $B-A$. From the result, we can easily find that the part between $O-A$ and $B-A$ are cut. Then do that again and again, it can be close to the desired trajectory finally.

The x position and force errors are shown in Fig. 8, and Fig. 9 respectively. From these figures, we can find the general tendency of position and force error is decrescent. And these errors are not only so tiny but also in the allowable range.

Δh shown in Fig. 10 means that the perpendicular distance from current position to desired position. After doing many times, if Δh can be close to zero, it means the shape grinding can be done very well. We show a combination of Δh each time in Fig. 10, it means the change process of Δh. In this simulation, we used a 9-th order polynomial, the terms of highter order than 9-th degree is omitted. So it caused the tiny error.

From Δh at the last time shown in Fig. 10, we can know maximum of Δh at the last time is less than 0.01[m] after 18 times grinding. And as time passes, Δh can be more smaller and less than 0.001[m] with about 50 times grinding.

Generally, although tiny errors exist, we can also say that shape grinding can be done very well by this method.

VI. Conclusions

The constrained dynamic equations of a manipulator are derived and the constrained forces are expressed as an explicit function of the state and inputs. The presented methodology allows computation of the forces, as an alternative to sensing. Hence, the system is controlled with no force sensor. The control law presented is constructed by using the dynamical redundancy of constrained systems. The controller designed with this control law can be used for simultaneous control of force and position. In the paper, we present three methods for estimating the constrained condition to attain time-varying unknown constrained information. Simulation results indicate that the method of "quadratic spline fitting for unknown constrained surface" has the best estimation of the real constrained surface. Hence we can say the performance of controller with quadratic spline fitting is the best.

Moreover, the quadratic spline fitting for unknown constrained surface is used in the shape grinding. From the last results, we can find that it can be done very well in the shaping-grinding.

References

[1] S. Arimoto, "Mechanics and Control of Robot(in Japanese)," Asakura Publishing Co., Ltd., Tokyo, Japan, 1990. T. Yoshikawa, "Dynamic Hybrid Position/Force control of Robot Manipulators — Description of Hand Constraints and Calculation of Joint Driving Force," IEEE J. on Robotics and Automation, Vol.RA-3, No.5, pp.386-392, 1987.

[2] L. Whitcomb, S. Arimoto, T. Naniwa and F. Osaki, "Experiments in Adaptive Model-Based Force Control," IEEE Control Systems Society, Vol.16, No.1, pp.49-57, 1996.

[3] Suguru Arimoto, "Control Theory of Non-Linear Mechanical Systems," Oxford University Press, 1996.

[4] B. Siciliano, L. Villani, "Robot Force Control," Kluwer Academic Publishers, U.K., 1999.

[5] C. Natale, "Interaction Control of Robot Manipulators," Springer Tracts in Advanced Robotics, Germany, 2003.